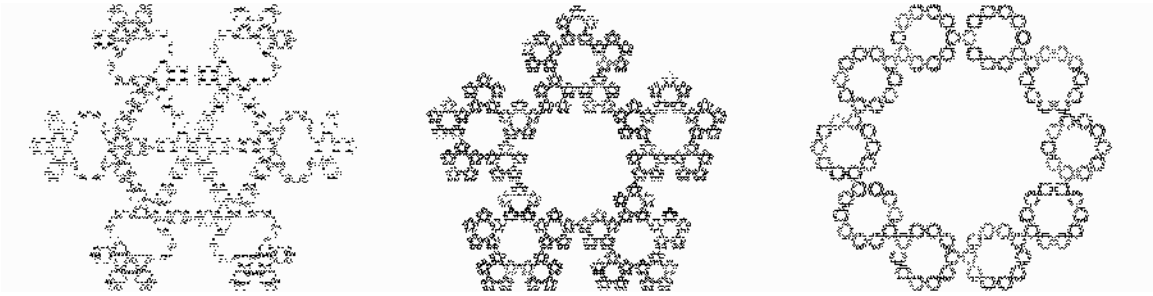
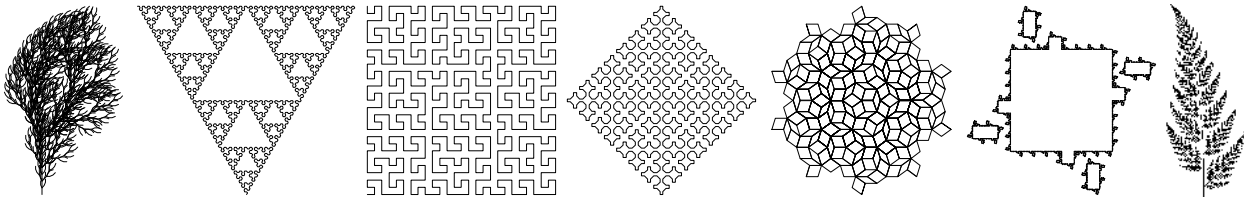
Name _____

L-Systems 1

These L-system activities are designed to help you create designs like the ones shown below.





Pretend that on the computer screen is a turtle facing towards the right. If you give the turtle the command '`F`', the turtle will crawl forward one unit and will leave a mark on the screen as it crawls. If you give the turtle the command '`G`', the turtle will crawl forward one unit, but will not leave a mark as it crawls.

So if the turtle sees the commands '`FGFF`', it will crawl forward *one* unit while leaving a mark. Then it will crawl forward *one* unit without leaving a mark. Then it will crawl forward *two* units while leaving a mark. So the resulting drawing will look like this: — ———

Normally, to make a drawing that will be interesting, we want the turtle to be able to turn to the left or the right. The command '`+`' tells the turtle turn to its left (i.e., counterclockwise). The command '`-`' tells the turtle turn to its right (i.e., clockwise). Before telling the turtle '`+`' or '`-`', we will tell the turtle how far to turn. The instruction '`Angle 4`' tells the turtle to turn $\frac{1}{4}$ of a circle (i.e., 90°) whenever it is told '`+`' or '`-`'. The instruction '`Angle 6`' tells the turtle to turn 60° whenever we command it to turn (since 360°÷6 = 60°). Suppose the turtle is told '`Angle 12`' and then is commanded '`F+F+F----F+F`'. It will 1st crawl forward one unit. Then it will turn counterclockwise 30° (since 360°÷12 = 30°). Then it will crawl forward a unit (so the drawing now looks like this: ). Then it will turn left 30° again and crawl forward (now the drawing looks like this:). Then it will turn right (i.e., clockwise) 4 times (for a total of 120°). Then it will crawl forward once (the drawing now looks like this: . Then it will turn counterclockwise 30° and crawl forward to finish the drawing.

Try to understand how the commands '`F+F+F----F+F`' produce this drawing: . To learn how to tell the computer to make designs, start the program *Arcnel*. Click *Edit* on the menu bar and then click *New L-system*. This displays the *L-system* window. After reading this paragraph, edit the text in the *L-system* window so that it contains the commands that follow this paragraph. The first line of a formula contains the name of the formula and a '`{`' (left brace). The brace tells the computer that the formula is now beginning. The next line tells the turtle how far to turn when it receives a '`+`' or '`-`' command. You do not yet need to understand the significance of the word '`Axiom`'. Just make sure it is in the L-system. The word '`Axiom`' is followed by the commands for the turtle. The formula ends with a '`}`' (right brace). To quickly change the name of the L-system, double-click '`New`' in the *L-system* window and type '`Sample1 `' (*Be sure to type the space at the end*. Do not type the quotes that surround what is to be typed). Edit the text in the *L-system* window until it contains these commands:

```
Sample1 {
  Angle 12
  Axiom F+F+F----F+F
}
```

Be sure that '`Sample1`' has *no space in it*. Put a space *after* '`Sample1`' and *before* the '`{`'.

Save this paper for future reference.

To have the computer process the `Sample1` L-system, click *Edit* on the menu bar and then click *Process Shown Text*. The *Arcnel* window should then draw this design:

To save the `Sample1` L-system, click *File* on the menu and then click *Save L-system To File*. Then type your initials followed by "`.AL`". For example, if you initials are PQR, then type `PQR.AL` for the file name. Then click the *Save* button.

Now select *New L-system* in the *Edit* menu and edit the text in the *L-system* window that appears so that it contains the commands that follow this paragraph. When you come to a '`;`' (semicolon), you do not need to type the rest of that line. For each line, *you do need to type all of the text that is before a semicolon*. **If a line does *not* contain a semicolon, then type *everything* on that line**.

```
Pentagon {         ; If the computer sees a semicolon, it ignores the rest of that line.
  Angle 5          ; 360/5 = 72. All text before a semicolon must be typed.
  Axiom F+F+F+F+F ; '+' = 'turn counterclockwise one-fifth of a circle'
}
```

To have the computer process the `Pentagon` L-system, select *Process Shown Text* in the *Edit* menu. The *Arcnel* window should then draw a pentagon. To save the `Pentagon` L-system, select *Save L-system To File* in the *File* menu. Then select the "`.AL`" file you saved above and click the *Save* button.

Now select *New L-system* in the *Edit* menu and edit the text in the *L-system* window that appears so that it contains the following commands:

```
Star { ; If text after a ';' isn't typed, then the ';' doesn't need to be typed either.
  Angle 10 ; 360/10 = 36. '-' = 'turn clockwise one-tenth of a circle'
  Axiom F----F----F----F----F
}
```

Select *Process Shown Text* in the *Edit* menu. The *Arcnel* window should then draw a 5-pointed star. Using the procedure taught above, now save the `Star` L-system to your "`.AL`" file.

To watch how the turtle draws the star, select *Debug* in the *Edit* menu. In the *Debug* window, click *Next* on the menu bar to have the turtle move one unit. Answer the following questions (you may write on this paper):

1) Click *Next* again. The leftmost triangle in the *Debug* window should have rotated. Point the mouse at that triangle. The computer should display an angle measure. What is that angle measure?
2) Click *Next* again. Point the mouse at the leftmost triangle in the *Debug* window. The computer should display an angle measure. What is that angle measure?
3) Click *Next*. Write the new angle measure.
4) Click *Next*. Write the new angle measure.
5) Click *Next*. The turtle should crawl forward one unit. Now click *Next* four times to make the turtle turn four times. Write the new angle measure.
6) Click *Auto*. Watch the turtle. Once the turtle is done moving, what is the angle measure?
7) Click *Next*. This exits *Debug* mode. Select *Open* in the *File* menu. Select the "`.AL`" file you saved and click the *Open* button. Click `Pentagon` in the list of L-systems. Click *OK*. An *Order* window should appear. A later activity teaches about the "Order" concept. For now, just click *OK* in the *Order* window. This should draw a pentagon. Select *Debug* in the *Edit* menu. A window on the screen should show the commands '`F+F+F+F+F`'. The title bar of that window should show the text "`Length:`" followed by a number. Write that number.
8) Click *Next* in the *Debug* window. The first '`+`' in the *Length* window should be selected. This means that '`+`' is the next command. Click *Next* to perform that command. Write the new angle measure.
9) Click *Auto*. Write the angle measure once the turtle stops moving. You may now exit from *Arcnel*.

Name _____

L-Systems 2

Start the program *Arcnel*. Click *Edit* on the menu bar and then click *New L-system*. Edit the text in the *L-system* window until it contains these commands:

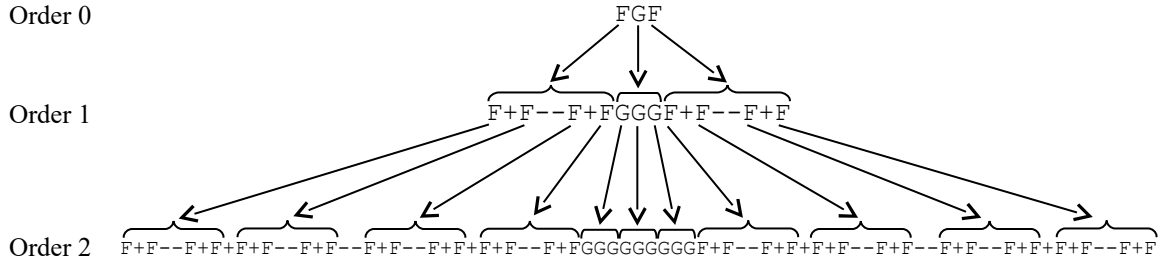
```
KochCurve2 { ; Remember to always type all text that precedes a semicolon.
  Angle 6    ; 360/6=60. '-' = 'turn clockwise one-sixth of a circle'
  Axiom FGF  ; Text after a semicolon doesn't need to be typed.
  F=F+F--F+F ; The lines after the axiom line contain substitution rules.
  G=GGG      ; Don't forget there are braces at the beginning and the end!
}
```

To have the computer process the `KochCurve2` L-system, select *Process Shown Text* in the *Edit* menu. Save the L-system as instructed in the first activity. Then select *L-system Order* in the *Edit* menu. In the window that pops up, type the digit 0 (zero). Click *OK*. Since the formula contains the command '`Axiom FGF`', the turtle draws a segment. Then it moves forward without leaving a mark. Then it draws another segment.

Now select *L-system Order* in the *Edit* menu. In the window that pops up, type the digit 1 (one). Click *OK*. Since the order is 1, the computer uses the substitution rules '`F=F+F--F+F`' and '`G=GGG`' to change the commands for the turtle. The substitution rule '`F=F+F--F+F`' tells the computer to replace each '`F`' in the axiom with '`F+F--F+F`'. Likewise, '`G=GGG`' tells the computer to replace the '`G`' in the axiom with the string '`GGG`'. So the computer replaces the original command string '`FGF`' with the command string '`F+F--F+FGGGF+F--F+F`'. The turtle uses this command string to draw the diagram. The computer automatically resizes the diagram so that it fits on the screen. To watch how the turtle draws this design, select *Debug* in the *Edit* menu.

1) A window on the screen should show the commands '`F+F--F+FGGGF+F--F+F`'. The title bar of that window should show the text "Length:" followed by a number. Write that number.
2) Click *Next* in the *Debug* window. Write the symbol that is selected in the *Length* window.
3) Click *Next* in the *Debug* window. Point the mouse at the leftmost triangle in the *Debug* window. What is the angle measure?

Click *Exit* in the *Debug* window. If the order were '2', the computer would perform the substitutions a second time. The text that results at each step is shown in the following diagram. Each time, the computer replaces each '`F`' with '`F+F--F+F`' and replaces each '`G`' with '`GGG`'.



In this example, replacing '`F`' with '`F=F+F--F+F`' is equivalent to replacing each straight segment with the shape ⎽⎽/\⎽⎽. Change the order to 2 and see the substitution occur. Then change the order to 3 and see the substitution occur again.

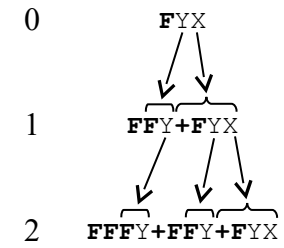
Click *Edit* on the menu bar and then click *New L-system*. Edit the text in the *L-system* window until it contains these commands:

```
TriSpiral {
  Angle 3 ; 360/3 = 120. '+' = 'turn counterclockwise one-third of a circle'
  Axiom FYX ; As the turtle crawls, it ignores symbols it doesn't understand.
  Y=FY      ; When the turtle sees a Y, the turtle ignores it.
  X=+FYX
}
```

This formula contains symbols '`X`' and '`Y`'. The turtle does not understand these symbols, so it ignores them. But as the computer computes orders 1, 2, 3, etc., it follows the substitution rules '`Y=FY`' and '`X=+FYX`'.

Do not discard this activity until after completing the rest of the activities.

Select *Process Shown Text* in the *Edit* menu. Save the L-system. Select *L-system Order* in the *Edit* menu. Change the order to 0 and click *OK*. Also try orders 1, 2, 3, and 14. Select *Curve Width* in the *Edit* menu. Type the digit 3 and click *OK*.



Substitution Rules
Y=**F**Y
X=**+F**YX

Sometimes, to increase understanding of an L-system, it can help to have the turtle color the segments it draws. Edit the text in the *L-system* window until it contains these commands:

```
TriSpiralColor { ; In the default color palette, color 15 is white.
  Angle 3
  Axiom C15FYX ; 'C15' tells the turtle to chose color 15.
  Y=>1FY       ; '>1' tells the turtle to decrease the color number by 1.
  X=+C15FYX    ; After each turn, chose color 15 again.
}
```

IMPORTANT NOTICE
If you need to change an L-system after saving it, open the ".AL" file in *Notepad* and edit that file and then save it. Then select *Open* in the *File* menu in *Arcnel* to reopen the changed ".AL" file.

Select *Process Shown Text* in the *Edit* menu. Save the L-system. Try orders 1, 2, 3, and 4 to see how the turtle colors different segments. Then try order 14. Notice how the first segment at the bottom-left is white (color 15). The next segment is color 14. The next segment is color 13, etc. Then when the turtle turns 120°, it chooses color 15 (white) again. To see the color palette, select *Show Editable Color Map* in the *Color* menu.

4) Point the mouse at the white square in the color palette. The computer should show *three* lines of information. The *first* line shows that the color number is 15. Write the entire *second* line of information.

5) The second line of information shows the red, green, and blue values for the color. Move the mouse around until you find color # 14. What are the red, green, and blue values for color 14?

Change the order to 3. For <u>*each one*</u> of the following *three* L-systems, first edit the *L-system* window so that it contains the L-system. Then select *Process Shown Text* in the *Edit* menu. Then save the L-system.

```
Snow {
  Angle 6
  Axiom F+F+F+F+F+F
  F=F+FF+F--F--F+FF+F
}
```

```
SnowColor {
  Angle 6 ; '<2' --> increase color # by 2
  Axiom C2F+<2F+<1F+<2F+<4F+<2F
  F=F+FF+F--F--F+FF+F
}
```

```
SnowColor2 {
  Angle 6 ; '>2' --> decrease color # by 2
  Axiom F+F+F+F+F+F
  F=C13F+>2F>2F+>2F-->1F-->1F+>1F>1F+>1F
}
```

The `Snow` design should look similar to the top-left design on the "L-Systems 1" activity. If you saved a flawed L-system, see the IMPORTANT NOTICE above.

6) Select *Open* in the *File* menu. Select the ".AL" file you saved. Choose `Snow` in the L-system list and click *OK*. Type 0 for the order and click *OK*. Name the shape produced.

7) Change the order to 1. Choose *Show Command String Text* in the *Edit* menu. The title bar of the *Length* window shows the length of the command string text. What is that length?

8) Select *Choose L-system* in the *File* menu. Choose `SnowColor2` in the L-system list and click *OK*. Make sure the order is 1 and click *OK*. Select *Show Command String Text* in the *Edit* menu. What is the length displayed in the title bar of the *Length* window?

9) Select *Choose L-system* in the *File* menu. Choose `SnowColor` in the L-system list and click *OK*. Make sure the order is 1 and click *OK*. Select *Debug* in the *Edit* menu. What is the length displayed in the title bar of the *Length* window?

10) Click *Next* in the *Debug* window. Remember the new color of the turtle. Now continue to click *Next* until the turtle turns a different color. What is the measure of the angle for the turtle?

11) Click *Exit* in the *Debug* window. Change the order to 4. Choose *Show Command String Text* in the *Edit* menu. A window should pop up. **Don't click any buttons**. According to the window that pops up, how long is the command string?

12) Click *No*. Change the order to 3. Choose *Show Command String Text* in the *Edit* menu. How long is the command string?

Name _____

L-Systems 3

Start *Arcnel*. Select *New L-system* in the *Edit* menu. For <u>each</u> of the following three L-systems, type the entire L-system in the *L-system* window and then process the text and then save the L-system.

```
KochCurve3 {
  Angle 6
  Axiom F!F!F
  F=F+F--F+F
}
```

```
KochCurve3Color {
  Angle 6
  Axiom C5FC7!F!C11F
  F=F+F--F+F
}
```

```
Zigzag {
  Angle 3
  Axiom F
  F=F!-F
}
```

The command '`!`' tells the turtle to <u>reverse</u> the meanings of '`+`' and '`-`'. In the axiom '`F!F!F`', the command '`!`' has no effect (since there are no '`+`' or '`-`' commands). But after the computer substitutes '`F+F--F+F`' for '`F`', the command '`!`' does have an effect. After applying the substitution rule '`F=F+F--F+F`' to the axiom '`F!F!F`', the command string is '`F+F--F+F!F+F--F+F!F+F--F+F`'. Normally, '**+**' means '**turn counterclockwise**' and '**-**' means '**turn clockwise**'. After the turtle sees the first '`!`', then '`+`' means '<u>turn clockwise</u>' and '`-`' means '<u>turn counterclockwise</u>'. When the turtle sees the next '`!`', it again reverses the meanings of '`+`' and '`-`'. So now they are back to their original meanings.

Open the file to which you saved the above three formulas and choose `KochCurve3` and click *OK*. Type the digit 1 for the order and click *OK*. Then select *Debug* in the *Edit* menu. Notice that an '`N`' is displayed in the *Debug* window. This means that '`+`' and '`-`' have their **normal** meaning. Click *Next* in the *Debug* window until the first '`!`' is selected in the *Length* window. The *Debug* window should still show an '`N`'.

1) Click *Next* one more time. Write the letter that is now displayed instead of the '`N`'.
2) Click *Next* two more times. What is the angle measure for the turtle?
3) Click *Next* in the *Debug* window until the second '`!`' is selected in the *Length* window. Then click *Next* three more times. What is the angle measure for the turtle?
4) Click *Exit*. Select *Choose L-system* in the *File* menu. Choose `KochCurve3Color`, click *OK*, type the digit 2 for the order, and click *OK*. Select *Debug* in the *Edit* menu. Observe the '`N`' in the *Debug* window. Click *Auto* in the *Debug* window and watch both the '`N`' and the turtle. What is the color of the design when the meanings of '`+`' and '`-`' are reversed?
5) Click *Exit*. Select *Choose L-system* in the *File* menu. Choose `Zigzag`, click *OK*, type the digit 4 for the order, and click *OK*. Select *Debug* in the *Edit* menu. Click *Auto*. When the turtle is finished moving, what is the meaning of '`-`'? (Use *words* in the answer. Read the top paragraphs on this page if you have questions.)

Click *Exit*. Enter the following L-system, process it, and save it.

IMPORTANT NOTICE
To delete an L-system after saving it, open the ".AL" file in *Notepad* and delete the entire L-system (including the name and both braces). Then save the file. Then select *Open* in the *File* menu in *Arcnel* to reopen the changed ".AL" file.

```
OctSpiralColor {
  Angle 8
  Axiom C16F
  F=F<1+@.99F ;'@' is the 'at' symbol which is found on the key for '2' on many keyboards
}
```

The command '`@.99`' tells the turtle to crawl $\frac{99}{100}$ as far as it used to crawl when it saw '`F`' or '`G`'. For order 2 (i.e., after 2 substitutions), the command string is '`C16F<1+@.99F<1+@.99F<1+@.99F`'. For the 1st '`F`', the turtle crawls the normal distance. For the 2nd '`F`', the turtle crawls 0.99 of the original distance. For the 3rd '`F`', the turtle crawls 0.99 of that new distance which means that it crawls 0.9801 of the original distance.

6) Do order 8 of `OctSpiralColor`. Select *Spectrum Map* in the *Color* menu. Select *Debug* in the *Edit* menu. Observe the number 1 displayed in the "Length" column in the *Debug* window. Click *Next* until that number changes. Write the new number.

Hint: Instead of clicking *Next* in the *Debug* window, you may press N on the keyboard if that window is active. Also, you may press A instead of clicking *Auto*, and you may press Esc instead of clicking *Exit*.

7) Click *Next* until the number changes again. Write the new number.

8) Click *Next* until the number changes again. Write the new number.

Click *Exit*. **Change the order to 3**. Select *Arcnel Map (Default)* in the *Color* menu. Select *New L-system* in the *Edit* menu. For *<u>each</u>* of the following three L-systems, type the L-system in the *L-system* window and then process the text and then save the L-system. The '|' symbol may look like '¦' on the keyboard.

```
KochAlt2 {
  Angle 6
  Axiom F|F
  F=F+F--F+F
}
```

```
KochAlt6 {
  Angle 6
  Axiom F|FF|F+F|FF|F+F|FF|F
  F=F+F--F+F
}
```

```
KochAlt6Color {
  Angle 6
  Axiom C2F|F<2F|F+<1F|F<2F|F+<2F|F<2F|F
  F=F+F--F+F
}
```

If the '`Angle`' command is an even number, then the '`|`' command tells the turtle to turn 180°. So in `KochAlt2`, the axiom '`F|F`' is equivalent to '`F+++F`'. At order 0, '`F|F`' causes the turtle to crawl forward, then turn around and crawl back the way it came. So at order 0, the extra '`F`' is not visible. At order 1 (after '`F+F−F+F`' is substituted for each '`F`' in '`F|F`'), the command will be '`F+F--F+F|F+F--F+F`'.

9) Open the file to which you saved the above 3 formulas (Be sure to select *Open* instead of *Choose L-system* in the *File* menu). Select `KochAlt2` and let the order be 1. How many line segments are visible in the design?

10) Change the order to 2. The design should have 3 polygonal regions. How many sides does the central polygon have?

11) Change the order to 3. How many polygonal regions are in the design?

12) Select *Choose L-system* in the *File* menu. Choose `KochAlt6` and let the order be 1. Select *Debug* in the *Edit* menu. Click *Auto*. When the turtle is finished moving, what is its angle measure?

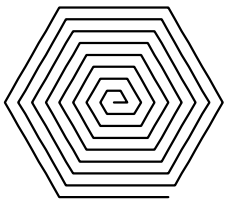
13) Click *Exit*. Change the order to 5. Select *Show Command String Text* in the *Edit* menu. What is the length of the text?

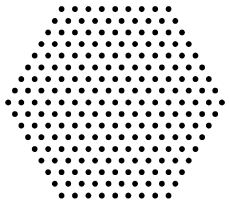
14) Select *Choose L-system* in the *File* menu. Choose `KochAlt6Color` and let the order be 2. Select *Debug* in the *Edit* menu. Click *Next* until the turtle is at the rightmost point in the design. Click *Next* one more time. The turtle should have changed its direction. By how many degrees did it change its direction?

15) Keep clicking *Next* until the turtle changes its color. What is its new color?

16) Keep clicking *Next* until the turtle changes its color again. What is its new color? (Make sure that the answers to #15 and #16 are different. If they are the same, you probably need to change the answer for #15)

17) Click *Exit*. Press F1 on the keyboard. The *Arcnel Help* window should appear. According to the information in it, who introduced L-systems?





```
HexagonalWalk {
  Angle 6
  Axiom Y
  Y=YF+X+XF+XF+XF+XF+XF
  X=XF
}
```

Both designs to the left were made using order 8 of `HexagonalWalk`. The first one was made using a width of 7. For the other, the width was changed to 17, and *Dot Mode* was selected in the *Edit* menu.

IMPORTANT NOTICE

To temporarily "delete" an L-system after saving it, open the ".AL" file in *Notepad* and type a semicolon at the beginning of *each* line of the L-system (including the name and the ending brace). Then save the file. This causes *Arcnel* to ignore those lines since a semicolon indicates the beginning of a comment. Those lines have been "commented out."

Do not discard this activity until after completing the rest of the activities.

Name _____

L-systems 4

Start *Arcnel*. Select *New L-system* in the *Edit* menu. For <u>*each*</u> of the following three L-systems, type the entire L-system in the *L-system* window and then process the text and then save the L-system.

```
DoubleSpiral {
  Angle 40
  Axiom ----[F]|F
  F=F+@.98F
}
```

```
FlakeV {
  Angle 5
  Axiom F+F+F+F+F
  F=F+F[+F]--F+F
}
```

```
FlakeVColor {
  Angle 5
  Axiom C2F+<2F+<1F+<2F+<4F
  F=F+F[+F]--F+F
}
```

The '`[`' command tells the turtle to remember its current condition (which includes its position, direction, color, length of crawl, and meaning of '+' and '-'). Then when the turtle is given the '`]`' command, it goes back (without leaving any marks) to the condition it had been in when it was given the '`[`' command.

1) Open the file to which you saved the above formulas. Choose `DoubleSpiral` and let the order be 5. Choose *Debug* in the *Edit* menu. Click *Next* until the '`[`' is selected in the *Length* window. Then click *Next* again. The *Debug* window should now show two lines of information. Look at the two leftmost triangles in the "Angle" column. What is the angle of the top triangle?

2) What is the angle of the bottom triangle?

3) Click *Next* until the '`]`' is selected in the *Length* window. Now what is the angle of the top triangle?

4) What is the angle of the bottom triangle?

5) The "Length" column in the *Debug* window shows the crawl length for the turtle. Write both the current crawl length (top line) and the remembered crawl length (bottom line) with as much precision as is displayed in the *Debug* window.

6) While watching the turtle, click *Next*. The remembered conditions have now become the current conditions. Click *Next* again. What is the current angle measure for the turtle?

7) Click *Next* two more times. By how many degrees did the angle measure change?

8) Click *Exit*. Select *Choose L-system* in the *File* menu. Choose `FlakeV` with an order of 1. How many pentagons are in the design?

9) Change the order to 5. Select *Show Command String Text* in the *Edit* menu. A window should pop up. Carefully read it, but don't click any buttons. According to that window, what is the length of the command string?

10) Click *No*. Select *Choose L-system* in the *File* menu. Choose `FlakeVColor` with an order of 2. Choose *Debug* in the *Edit* menu. Click *Next* until the first '`[`' is selected in the *Length* window. Then click *Next* until the first '`]`' is selected. Then click *Next* until the 2nd '`[`' is selected. Then click *Next* until the 2nd '`]`' is selected. Then click *Next* until the 3rd '`[`' is selected. Then click *Next* until the 4th '`[`' is selected. Click *Next* three more times. The *Debug* window should show 3 lines of information. The left triangle in the 1st line shows the current angle of the turtle. The left triangle in the 2nd line shows the most recently remembered angle. The left triangle in the 3rd line shows an angle that had been remembered earlier. Write all three angle measures, starting with the current angle.

11) The *Arcnel* window shows two small circles drawn on the design. The one circle is larger than the other. The smaller circle shows the most recently remembered location. The larger circle shows an earlier remembered location. Carefully watch the larger circle as you click *Next*. What happens to that circle?

12) Click *Next* until the turtle turns green. What is its angle measure?

13) Click *Exit*. Change the order to 5. Observe how the colors are distributed around the design. Studying colors may increase understanding regarding how the turtle crawls. Press F1. Find the "Push and Pop" section in *Arcnel Help*. According to the information in that section, what is the '`]`' command called?

Name _____

L-Systems 5

Start *Arcnel*. Select *New L-system* in the *Edit* menu. For <u>*each*</u> of the following two L-systems, type the L-system in the *L-system* window and then process the text and then save the L-system.

```
KochSnow {
  Angle 12
  Axiom +F----F----F
  F=F++F----F++F
}
```

```
KochSnowBuild {
  Angle 12
  Axiom +FY----FY----FY
  F=    ;This is not a mistake
  X=FY
  Y=FX[F]++FX----FX++FX
}
```

The Koch snowflake is named after Helge von Koch. *KochSnow* can help you visualize the design.

1) Open the file to which you saved the above formulas. Choose `KochSnow` and let the order be 0. How many line segments are in the design?

2) Change the order to 1. How many line segments are in the design?

3) Change the order to 2. How many line segments are in the design?

4) Change the order to 5. This should give you some idea of how the Koch snowflake would look. An infinite order would be needed to produce the actual Koch snowflake. Select *Show Command String Text* in the *Edit* menu. What is the length of that text?

5) Select *Choose L-system* in the *File* menu. Choose `KochSnowBuild` and let the order be 0. This L-system contains the substitution rule '`F=`' (with nothing after '`=`'). This rule will replace '`F`' with nothing (i.e., it will delete each '`F`'). This L-system shows one way of visualizing the construction of the Koch snowflake. What special type of triangle is displayed when the order is 0?

6) Change the order to 1. This places an equilateral triangle on the middle third of each side of the original triangle. Now change the order to 2. This should erase the boundaries between the original triangle and the little triangles. Now change the order to 3. How many little triangles does this add to the design?

7) Change the order to 4. This should erase the boundaries between the main design and the little triangles. Change the order to 5. Select *Debug* in the *Edit* menu. Click *Next* until the first '`]`' is selected in the *Length* window. Write both the current angle measure for the turtle and the remembered angle measure.

8) Click *Next* three times. What is the current angle measure?

Click *Exit*. Select *New L-system* in the *Edit* menu. For <u>*each*</u> of the following two L-systems, type the L-system in the *L-system* window and then process the text and then save the L-system.

```
QBlock2 {
  Angle 8
  Axiom F++F++F++F
  F=F[++F]G[@Q8+F]G++G|F++F
  G=GGGG ;4 G's
}
```

```
Quilt2 {
  Angle 8
  Axiom F++F++F++F
  F=F[++F--F]@I2G@Q2+F++F++F++F@IQ2+G@2[++!F]F
  G=GGG ;3 G's
}
```

9) In `QBlock2`, the command '`@Q8`' multiplies the turtle's crawl length by the square root of 8. In `Quilt2`, the command '`@I2`' multiples the crawl length by the inverse of 2 (i.e., $\frac{1}{2}$). The command '`@IQ2`' multiplies the crawl length by the inverse of the square root of 2. Do order 1 of `Quilt2`. How many squares are tilted?

10) The commands for drawing tilted squares are found between the commands '`@Q2`' and '`@IQ2`'. The command '`@IQ2`' restores the crawl length to what it had been before the '`@Q2`' command. Select *Debug* in the *Edit* menu. Click *Next* until the first '`@I2`' is selected. What is the current crawl length?

11) Click *Next* once. What is the new crawl length?

12) Click *Next* twice. Write the new crawl length rounded to three decimal places.

13) Click *Next* until the first '`@2`' is selected. What is the current crawl length?

14) Click *Next* once. What is the new crawl length?

15) Click *Exit*. Change the order to 5. Select *Show Command String Text* in the *Edit* menu. A window should pop up. Do not click any buttons. According to the window that pops up, what is the length of the command string?

16) Click *No*. Open the file to which you saved the formulas. Choose `QBlock2` and let the order be 1. Select *Debug* in the *Edit* menu. The command for drawing one side of the large tilted square is the '`F`' in '`[@Q8+F]`'. Notice that at the '`]`' command, the turtle returns to the condition in which it had been when it received the '`[`' command (including the size of the crawl). So the command '`@IQ8`' was not needed to restore the crawl length to its original size. How many squares are in the design?

17) Click *Next* until the first '`@Q8`' is selected. What is the crawl length?

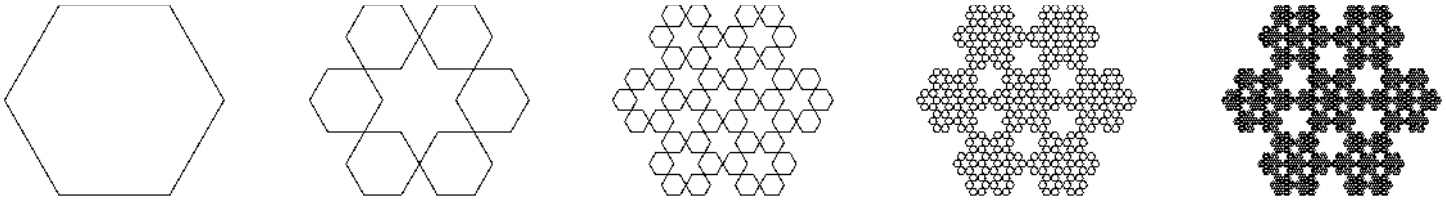
18) Click *Next* once. Write the current crawl length rounded to 3 decimal places.

19) Click *Next* twice. What is the current angle measure?

20) Click *Next* once. Write both the current angle and crawl length.

21) Click *Exit*. Change the order to 2. How many tilted squares are in the design?

22) Study the central square in the order 2 diagram. Then study the central squares for orders 3, 4, and 5. How does the central square in an odd order compare to the central square in an even order?



The above designs are orders 0 through 4 of the L-system shown below.

```
Flake6Fill {
  Angle 6
  Axiom F+KF+KF+KF+KF+KF+K
  K=@I3F+KF+KF+[|K]KF+KF+KF+K@3
  F=G
}
```

Name _____

L-Systems 6

Start *Arcnel*. Select *New L-system* in the *Edit* menu. For *each* of the following two L-systems, type the L-system in the *L-system* window and then process the text and then save the L-system.

```
FlakeIXb {
  Angle 9
  Axiom F+F+F+F+F+F+F+F+F
  F=F+F+F[+F+F+F]|F+F+F
;For Angle 9, '|'='+++++' for default meaning of '+'
}
```

```
MultiSpiral6G {
  Angle 40
  Axiom ----[FX]|FX
  X=[@.35Y][|@.35Y]+@.98F+@.98FX
  Y=++@.964FY
}
```

1) If the # in the command '`Angle #`' is even, then '`|`' tells the turtle to turn 180°. If the # is odd, then the turtle will turn counterclockwise 180° and will keep turning until it reaches a multiple of 360° ÷ #. For '`Angle 9`', 360° ÷ 9 = 40°. Since 4(40°) = 160° and 5(40°) = 200°, '`|`' tells the turtle to turn counterclockwise 200° (equivalent to turning clockwise 160°). So when '`+`' and '`-`' have their normal meanings, the '`|`' is equivalent to '`+++++`' which is equivalent to '`----`'. Open the file to which you saved the above formulas. Choose `FlakeIXb` and let the order be 1. Select *Debug* in the *Edit* menu. Click *Next* until the first '`|`' is selected in the *Length* window. What is the current angle measure for the turtle?
2) Click *Next* once. What is the current angle measure for the turtle?
3) Click *Next* until the next '`|`' is selected in the *Length* window. What is the current angle measure for the turtle?
4) Click *Next* once. By how many degrees did the angle measure change?
5) Click *Exit*. `MultiSpiral6G` was derived from the `DoubleSpiral` formula of a previous activity. Order 8 works well for `DoubleSpiral`, but `MultiSpiral6G` needs a much higher order. Select *Choose L-system* in the *File* menu. Choose `MultiSpiral6G` with an order of 88. Select *Show Command String Text* in the *Edit* menu. According to the window that pops up, what is the length of the command string?
6) Click *No*. Change the order to 3. Select *Debug* in the *Edit* menu. Click *Next* until the first '`]`' is selected in the *Length* window. What is the current angle measure for the turtle?
7) Click *Next* once. What is the current angle measure for the turtle?
8) Click *Next* until the next '`]`' is selected in the *Length* window. What is the current angle measure for the turtle?
9) Click *Exit*. Open "Arcnel.AL" (If it doesn't exist, click *Cancel* to close the *Open* dialog box. Select *Write Arcnel.AL* in the *File* menu. Click the *Save* button. Then open "Arcnel.AL"). Choose `FlakeX` and let the order be 4. Then set the order to 1. The design has 10 congruent regular polygons joined together. Write the name for those polygons.
10) Open "Fractint.L" (If it doesn't exist, select *Write Fractint.L* in the *File* menu and click the *Save* button. Then open "Fractint.L"). Choose `Bush` and let the order be 4. Select *Show L-system Text* in the *Edit* menu. Write the axiom (If you don't know what the axiom is, press F1 and read *Arcnel Help* until you understand).
11) Select *Choose L-system* in the *File* menu. Choose `Curve4` and let the order be 6. Also try orders 7, 8, and 9. Concerning the overall design, how do the odd and even orders differ from each other?
12) Now choose `Fass1` and let the order be 4. Then choose `Peano2` and let the order be 4. Select *Show L-system Text* in the *Edit* menu. Write the axiom.
13) Now choose `Penrose4` and let the order be 4. Select *Show L-system Text* in the *Edit* menu. Write the axiom.
14) Now choose `Island1` and let the order be 2. The design consists of three sizes of islands. There is one large island. How many medium islands are in the design?

If you develop any interesting designs, the instructor would be interested in seeing the formulas and pictures.

Name _____

L-Systems 7

Observe that each of the following L-systems has a *tilde* (~) immediately after the opening brace. This tilde tells *Arcnel* that an L-system is an **arc L-system**. In an arc L-system, '`L`' tells the turtle to draw an arc of a circle while turning to the left (i.e., counterclockwise), and '`R`' tells the turtle to draw an arc of a circle while turning to the right (i.e., clockwise). Start *Arcnel*. Select *New L-system* in the *Edit* menu. For *<u>each</u>* of the following four L-systems, type the L-system in the *L-system* window and then process the text and then save the L-system.

```
Rapids {~
  Angle 5
  Axiom A
  A=RLA
}
```

```
Spiral4c {~
  Angle 4
  Axiom C16L
  L=L@.9<1L
}
```

```
Spirals8c {~
  Angle 8
  Axiom C16Y
  X=@.93LX
  Y=+@.9[@.4LX]G<2Y
}
```

```
Spirals9c {~
  Angle 9
  Axiom C16KZ
  K=@.9LKR@I.9
  Z=<1@.9LKZ
}
```

1) Open the file to which you saved the above formulas. Choose `Rapids` and let the order be 4. Select *Debug* in the *Edit* menu. What is the length of the command string? How many times does '`L`' appear in the command string?

2) Click *Next* once. What is the current angle measure for the turtle? Study the arc traced by the turtle. Did the turtle turn clockwise or counterclockwise as it made the arc? By how many degrees did the turtle turn as it made the arc? Hint: The initial angle of the turtle can be thought of as being 360°.

3) Click *Auto*. What is the final angle measure for the turtle?

4) Click *Exit*. Select *Choose L-system* in the *File* menu. Choose `Spiral4c` with an order of 5. Select *Debug* in the *Edit* menu. What is the length of the command string?

5) Carefully watch the turtle. Click *Next* once. What happened to the turtle when *Next* was clicked?

6) Click *Next* again. Did the turtle turn clockwise or counterclockwise as it made the arc? By how many degrees did the turtle turn?

7) Click *Next* until the third '`L`' is selected in the *Length* window. When the turtle draws an arc, it uses its linear crawl length for the radius of the circle that contains the arc it will draw. What is the current crawl length for the turtle?

8) Click *Auto*. When the turtle is done crawling, write the displayed crawl length to 5 decimal places.

9) Click *Exit*. Select *Choose L-system* in the *File* menu. Choose `Spirals8c` with an order of 50. Press W on keyboard and change the width to 3. The design should show a spiral of spirals. If we start counting with the biggest spiral at the bottom of the window and go counterclockwise, the 3rd spiral should be yellow and the 4th spiral should be green. Which spiral is the next yellow spiral? Which spiral after that one is the next yellow spiral?

10) Select *Debug* in the *Edit* menu. Click *Next* until the first '`]`' is selected in the *Length* window. Write the current crawl length rounded to 6 decimal places.

11) Click *Next* two more times. What is the current angle measure for the turtle?

12) Click *Exit*. Select *Choose L-system* in the *File* menu. Choose `Spirals9c` with an order of 70. The design should show a spiral of connected spirals. Select *Debug* in the *Edit* menu. What is the length of the command string?

13) Click *Next* three times. What is the current angle measure for the turtle?

14) Look over the command string and find the first '`<1`'. Click *Next* once. Then press N on the keyboard until that first '`<1`' is selected. What is the current angle measure for the turtle?

Name _____

L-Systems 8

Arc L-systems have extra commands available that can be useful even when arcs don't need to be drawn. The extra commands '`&`', '`:`', '`` ` ``', '`%`', '`*`', and '`$`' are in the arc L-systems below. Don't confuse '`` ` ``' (grave character) with an apostrophe or single quote mark. Start *Arcnel*. **Change the order to 4**. Select *New L-system*. Type *each* L-system below in the *L-system* window, process the text, and save the L-system (`Leaf130b` should draw only a vertical line).

```
Leaf129c {~
  Angle 8
  Axiom ++A
  F=@1.36F@I1.36
  A=F[+:2&A][-:2&A]FA
}
```

```
Leaf130b {~
  Angle 8
  Axiom ++A
  F=@1.18F@I1.18
  A=F[+:4&A]F!A
}
```

```
CircleGrid {~
  Angle 4
  Axiom $XA$B
  A=:`GXA ;don't confuse ` and '
  X=[@.2G+LLLL]
  B=:%+G+X!G:%*1B
}
```

1) Open the file to which you saved the above formulas. Choose `Leaf129c` and let the order be 21. This draws a leaf shown on p. 129 of the book *The Algorithmic Beauty of Plants* (written by Przemyslaw Prusinkiewicz and Aristid Lindenmayer). The substitution rule '`F=@1.36F@I1.36`' makes use of a concept used in the `Leaf1` and `Leaf2` L-systems in the "Fractint.L" file. Select *Show Command String Text* in the *Edit* menu. According to the window that pops up, what is the length of the command string?

2) Click *No*. In arc L-systems, the '`&`' tells *Arcnel* to ignore the next character as *Arcnel* applies substitution rules. So the substitution rule '`A=F[+:2&A][-:2&A]FA`' is not used on an '`A`' that follows an '`&`' in the command string. For order 1 of `Leaf129c`, the command string contains '`:2`'. As *Arcnel* applies substitution rules to that command string to make the command string for order 2, it replaces '`:2`' with '`:1`' (the number after the colon is decreased by 1). When *Arcnel* applies substitution rules to make order 3, it replaces '`:1`' with '`:`' (which is equivalent to '`:0`'). When making order 4, *Arcnel* deletes '`:`' and the symbol after it (which will be '`&`' in this command string). Since the '`&`' before '`A`' is now deleted, *Arcnel* no longer ignores '`A`' and *Arcnel* applies '`A=F[+:2&A][-:2&A]FA`' to '`A`'. Change the order to 1. Select *Show Command String Text*. What is the total number of colons in the command string? Change the order to 2. Select *Show Command String Text*. What is the total number of colons in the command string? Change the order to 3. Select *Show Command String Text*. What is the total number of colons in the command string?

3) Change the order to 4. Select *Debug* in the *Edit* menu. Click *Auto* in the *Debug* window. As the turtle crawls, observe that no colons or ampersands are ever selected in the *Length* window since the turtle does not understand those symbols. How many colons are in the command string?

4) Click *Exit*. Select *Choose L-system* in the *File* menu. Choose `Leaf130b` with an order of 34. This draws a leaf shown on p. 130 of *The Algorithmic Beauty of Plants*. The algorithm in `Leaf130b` is shorter than the algorithm in the book since `Leaf130b` makes use of '`!`' which is not used in the algorithm in the book. Select *Show Command String Text*. What is the length of the command string?

5) Click *No*. Change the order to 6. Select *Debug* in the *Edit* menu. Click *Auto* in the *Debug* window. How many colons are in the command string?

6) Click *Exit*. Select *Choose L-system* in the *File* menu. Choose `CircleGrid` with an order of 5. Press W and change the width to 5 and click *OK*. How many circles are drawn?

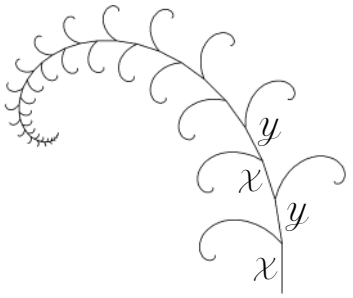
7) The turtle ignores a symbol immediately after a '`` ` ``' (grave character). When the turtle sees '`%`', it ignores all symbols up to the next '`%`'. Then it looks at symbols again. If it comes to another '`%`', it ignores all symbols up to the next '`%`'. Then it looks at symbols again, etc. As *Arcnel* applies substitution rules, it replaces '`*1`' with all the characters that were between the first pair of '`$`' symbols in the command string before the current round of substitution. Change the order to 1. Select *Debug* in the *Edit* menu. Click *Next* until '`]`' is selected in the *Length* window. Write the first three characters that immediately follow '`]`'.

8) Click *Next* twice. Change the order to 2. Select *Debug* in the *Edit* menu. Click *Next* until the first '`]`' is selected in the *Length* window. Write the character that immediately follows '`]`'.

9) Click *Auto*. Once the turtle is done crawling, click *Exit*. Change the order to 3. How many circles are drawn?

Name _____

L-Systems 9




```
SpiralPlant1 {
Angle 40
Axiom ++++++++++X
  F=@1.1F@I1.1
  X=F+[W]Y
  Y=F+[!W]X
  W=++++@0.3A
  A=F+A
}
```

```
SpiralPlant2 {
Angle 40
Axiom ++++++++++X
  F=@1.1F@I1.1
  X=F+[W]Y
  Y=F+[!W]X
  W=++++@0.3A
  A=F+[V]B
  B=F+[!V]A
  V=++++@0.3U
  U=F+U
}
```

```
SpiralPlant3c {~
Angle 40
Axiom +++++++X
  L=@1.1L@I1.1
  X=<1L[W]Y
  Y=<1L[!W]X
  W=++++@0.3A
  A=L[V]B
  B=L[!V]A
  V=++++@0.3U
  U=LU
}
```

Suppose a special plant grows as shown to the right. This plant has alternating sections. Call the bottom section an χ section and the one above it a y section. Then the next one is an χ section. The next one is a y section, etc. As a new section is added, it bends slightly to the left. At the top of an χ section, a small stem grows to the left of the main stem. At the top of a y section, a small stem grows to the right. Call $\mathcal{W}$ the point where a small stem attaches to the main stem. Call $\mathcal{A}$ the straight pieces of the small stems. At each stage of growth, the existing stems lengthen by a factor of 1.1. Start *Arcnel*. Select *New L-system* in the *Edit* menu. Type the `SpiralPlant1` L-system in the *L-system* window. Process the text and save the L-system. *Arcnel* should display 4 line segments (3 of which are in the main stem). The axiom '`++++++++++X`' turns the turtle so it crawls up instead of to the right. The axiom also places an χ section at the base of the plant. The '`F`' in the substitution rule '`X=F+[W]Y`' draws the portion of the main stem in an χ section. The '`+`' tells the turtle to turn 9° to the left. The '`[W]`' puts a $\mathcal{W}$ attachment point at the top of an χ section. Since '`W`' is in brackets, the small stem attached at a $\mathcal{W}$ point does not affect the rest of the plant. The '`Y`' places a y section as the next section in the main stem. Once the substitution rule '`W=++++@0.3A`' has been applied, the '`++++`' tells the turtle to turn 36° before it draws a small stem. The '`@0.3`' decreases the crawl length so a small stem will be a lot shorter than the main stem. The '`A`' causes a small piece of a small stem to be placed where the small stem attaches to the main stem. The substitution rule '`A=F+A`' causes the turtle to draw a small $\mathcal{A}$ piece of a small stem and then to turn 9°. The '`A`' at the end of the substitution rule places another small $\mathcal{A}$ piece after the piece that was just drawn. The substitution rule '`F=@1.1F@I1.1`' multiplies by 1.1 the lengths of existing pieces. Future pieces aren't changed. The '`[!W]`' in the substitution rule '`Y=F+[!W]X`' places a $\mathcal{W}$ attachment point at the top of a y section. Since this $\mathcal{W}$ attachment point is preceded by a '`!`', the turtle will turn the opposite directions it turns when drawing a small stem at the top of an χ section.

1) Change the order to 4. Two small stems should be attached to the main stem. Select *Debug* in the *Edit* menu. The title bar of the *Length* window should show a length of 126 (If not, click *Exit* and fix the L-system. Process the text and save the L-system. Then select *Debug* again). Click *Auto* in the *Debug* window. What is the total number of line segments drawn by the turtle?
2) Click *Exit*. Change the order to 5. How many small stems are attached to the main stem?
3) Change the order to 40. The design should look like the design shown above. To attach smaller stems to the small stems, edit the text in the *L-system* window so it is changed to the `SpiralPlant2` L-system shown above. Process the text and save the L-system. This changes the small stems so they consist of alternating $\mathcal{A}$ and $\mathcal{B}$ sections. At the end of each of these sections is a $\mathcal{V}$ attachment point for a smaller stem made of $\mathcal{U}$ sections. Select *Show Command String Text* in the *Edit* menu. What is the length of the command string?
4) Click *No*. Change the order to 7. Select *Debug* in the *Edit* menu. Click *Delay* in the *Debug* window and change the number to 100. Click *OK*. Click *Auto*. When the turtle is done crawling, what is its angle?
5) Click *Exit*. Edit the text in the *L-system* window so it is changed to the `SpiralPlant3c` arc L-system shown above. Observe that each '`F+`' in `SpiralPlant2` is replaced with the arc '`L`' command. Also, the base of the plant will not be vertical because of the changed axiom. Process the text. Save the L-system. Select *Debug* in the *Edit* menu. The length should be 671. Click *Auto*. When the turtle is done crawling, what is its angle?
6) Click *Exit*. Change the order to 40. Select *Show Command String Text* in the *Edit* menu. What is the length of the command string?
7) Click *No*. Change the order to 50. Select *Show Command String Text*. What is the length?
8) Click *No*. Select *Fern60* in the *Extra* menu. Press L. Find '`/`' in the L-system. Press F1. Find in the "More Angles" section where '`/`' is mentioned. What does '`/45`' tell the turtle to do when '`/`' has its normal meaning?

9) Select *SierpinskiCurveC* in the *Extra* menu. This draws a curve to approximate the Sierpinski triangle. Find both ends of the curve. At which vertex of the main triangle is the center of the curve?

10) Open "Arcnel.AL" (If it doesn't exist, select *Write Arcnel.AL* in the *File* menu. Click the *Save* button. Then open "Arcnel.AL"). Choose `SrpnskiFace` with an order of 1 (To find `SrpnskiFace`, scroll nearly to the end of the '`S`' section in the list of names). Select *Debug* in the *Edit* menu. Click *Next* many times while observing how many degrees the turtle turns at each '`+`'. Click *Auto*. When the turtle stops, at what vertex is it? What is its angle? How many '`+`' commands would be needed to change its angle to 0° (the same direction as 360°)?

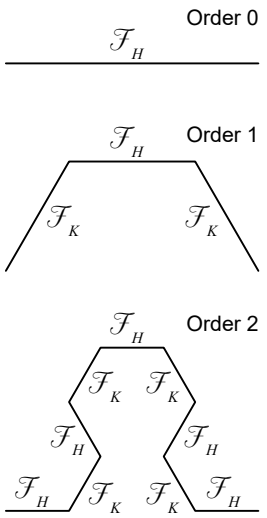
11) Click *Exit*. How many faces are drawn? Change the order to 2. How many faces are drawn? Change the order to 3. How many faces are drawn?

12) Select *Show L-system Text* in the *Edit* menu. Edit the name to change it to `SrpnskiFaceL`. The axiom is quite long. Make it even longer by adding `++C4FH` to the end of it. Add these two substitution rules to the L-system: `H=+FK-FH-FK+` and `K=-FH+FK+FH-`. Process the text. This should draw an orange curve consisting of line segments where each segment is on a side of a small triangle that contains a face. Save the L-system. Change the order to 1. Imagine the turtle starting at the left end of the orange curve and crawling up the first segment. Would a triangle with a face be on the left or the right side of the turtle? As the turtle crawls across the next orange segment, would a triangle with a face be on the turtle's left or right side? As the turtle crawls down the last orange segment, would a triangle with a face be on the left or the right side?

13) Change the order to 0. If the turtle starts at the left end of the orange segment and crawls the length of the segment, on which side of the turtle is there a triangle with a face?

14) Change the order to 2. If the turtle starts at the left end of the orange curve and crawls the length of the first orange segment, on which side of the turtle is there a triangle with a face? As it crawls the next segment, on which side of the turtle is there a triangle with a face? As it crawls the next segment, on which side of the turtle is there a triangle with a face?

15) Let $\mathcal{F}_H$ be an orange segment where there is a face on the *left* side of the turtle. Let $\mathcal{F}_K$ be an orange segment where there is a face on the *right* side of the turtle. The diagrams to the right label the line segments accordingly. When going from order 0 to order 1, $\mathcal{F}_H$ is replaced with $+\mathcal{F}_K-\mathcal{F}_H-\mathcal{F}_K+$ where '+' means to rotate 60° counterclockwise and '-' means to rotate 60° clockwise. When going from order 1 to order 2, $\mathcal{F}_K$ is replaced with $-\mathcal{F}_H+\mathcal{F}_K+\mathcal{F}_H-$, and $\mathcal{F}_H$ is again replaced with $+\mathcal{F}_K-\mathcal{F}_H-\mathcal{F}_K+$. This produces the sequence $+-\mathcal{F}_H+\mathcal{F}_K+\mathcal{F}_H--+\mathcal{F}_K-\mathcal{F}_H-\mathcal{F}_K+--\mathcal{F}_H+\mathcal{F}_K+\mathcal{F}_H-+$. These substitutions are implemented in `SrpnskiL` which is shown below the Order 2 diagram. Edit the text in the *L-system* window so it is changed to the `SrpnskiL` L-system. Process the text. This should draw the order 2 curve shown to the right. Save the L-system. Change the order to 3. How many line segments are in the curve?



16) Change the order to 4. Select *Debug* in the *Edit* menu. Click *Auto*. As the turtle crawls, observe that it does a lot of turning back and forth at certain vertices. Write the last 4 characters in the command string.

17) Click *Exit*. Except for the beginning of a curve, each $\mathcal{F}_H$ segment is preceded by an $\mathcal{F}_K$ segment, and each $\mathcal{F}_K$ segment is preceded by an $\mathcal{F}_H$ segment. Both '+' and '-' are added at a vertex when going from one order to the next. These rotations undo each other. To decrease the amount of turning, change the '`H`' and '`K`' rules to these: `H=FK-FH-FK` and `K=FH+FK+FH`. Change the name to `SrpnskiL2`. Process the text and save the L-system. The curve shouldn't change. At odd orders it changes orientation. Change the order to 3. Select *Debug* in the *Edit* menu. Click *Auto*. When the turtle is finished, what is its angle?

```
SrpnskiL {~
  Angle 6
  Axiom C4FH
  F=
  H=+FK-FH-FK+
  K=-FH+FK+FH-
}
```

18) Click *Exit*. Delete '`C4`' from the axiom. Change the '`H`' rule to `H=:1~<1~FK-FH-FK`. Change the name to `SrpnskiL2c`. Process the text and save the L-system. Change the order to 7. Select *Spectrum Map* in the *Color* menu. At which vertex of the main triangle is the center of the curve?

19) Select *Help* in the *Extra* menu. Find in the "Arcs" section the paragraph that talks about the '`#`' and '`~`' symbols. When one '`~`' is followed by another, what does the first '`~`' tell *Arcnel* to do?

Name _____

L-Systems 10

```
PlantD {~653
  Axiom ?24\90DK
  K=,3,1 ;2 choices
  K=[?B]!B,A
  B=\25\?10T
  A=\??20T
  T=@.8@?.7DK
}
```

The Creator designed plants so they can respond to conditions in the environment. These responses can result in many variations in plant design. From our perspective, these variations may appear random. We can approximately mimic these variations by introducing randomness into L-systems. Start *Arcnel*. Change the width to 3. Select *New L-system* in the *Edit* menu. Type the `PlantD` L-system in the *L-system* window (This arc L-system doesn't need an '`Angle`' command since it doesn't have '`F`', '`G`', '`L`', '`R`', '`+`', '`-`', or '`|`' and doesn't make use of the angle number specified by an '`Angle`' command). Process the text. *Arcnel* should display a vertical stem with two branches. Save the L-system. Select *Debug* in the *Edit* menu. The text '`?24`' should be selected in the *Length* window. The title bar of the *Length* window should show a length of 43 (If not, click *Exit* and fix the L-system. Process the text and save the L-system. Then select *Debug* again). Point the mouse at the turtle and wait a little bit. A window should appear. In that window, the bottom line of information shows the value of a special number called a *seed*. The seed contains information about the state of a "random" number generator. The "random" number generator produces a sequence of numbers with the superficial appearance of randomness. However, a precise formula generates the numbers, so they are not truly random. They are pseudorandom. The command '`?24`' tells the turtle to set the state of the pseudorandom number generator. Click *Next*.

1) Point the mouse at the turtle. According to the window that appears, what is the value of the seed?
2) The text '`\90`' should be selected in the *Length* window. This tells the turtle to turn 90° counterclockwise. The "Mode" column in the *Debug* window should display '`N F`'. Click *Next*. What does the "Mode" column now display?
3) The text '`D`' should be selected in the *Length* window. This tells the turtle to draw while crawling. The commands '`\`' and '`/`' change the direction the turtle crawls when it sees '`D`'. The commands '`+`' and '`-`' change the direction the turtle crawls when it sees '`F`'. The first triangle in the *Debug* window should point to the right. That is the direction the turtle would crawl if it saw an '`F`'. The second triangle should point up. That is the direction for a '`D`' command. Click *Next* twice. The text '`?`' should be selected in the *Length* window. This tells the turtle to ask the pseudorandom number generator for a number that will be ignored. This changes the state of the generator. Point the mouse at the turtle and observe that the seed has not changed. Click *Next*. Point the mouse at the turtle. What is the seed?
4) The text '`\25`' should be selected in the *Length* window. Watch the triangles in the *Debug* window. Click *Next*. One triangle should change direction. It shows the angle of the turtle when it is in '`D`' mode. Point the mouse at that triangle. What is the angle of the turtle?
5) The text '`\?10`' should be selected in the *Length* window. This tells the turtle to turn counterclockwise a "random" angle from 0° up to (but not including) 10°. Click *Next*. To the nearest hundredth, what is the angle of the turtle? What is the value of the seed?
6) Click *Next*. The text '`@?.7`' should be selected in the *Length* window. This tells the turtle to pick a "random" multiplier from 1 down to (but not including) 0.7. Click *Next*. The "Multiplier" column shows the multiplier picked by the turtle. Click *Next* twice. The text '`!`' should be selected. This will reverse the meanings of '`\`' and '`/`'. Click *Next*. The "Mode" column should display '`R D`', and '`\25`' should be selected in the *Length* window. Click *Next*. What is the angle of the turtle? What is the value of the seed?
7) The text '`\?10`' should be selected in the *Length* window. Since '`\`' and '`/`' have reversed meanings, the turtle will turn clockwise. Click *Next*. To two decimal places, what is the angle of the turtle?
8) Click *Exit*. Change the order to 12. The right and left sides of the plant have different structures. This is a result of the '`K`' substitution rule which is written on two lines as `K=,3,1` and `K=[?B]!B,A`. The first line has a comma after the '='. This tells *Arcnel* that the '`K`' substitution rule contains options. The two numbers tell *Arcnel* that there are two options which are to be chosen in a probability ratio of 3:1. The second line lists the two options separated by a comma. Around 75% of the time, *Arcnel* replaces '`K`' with '`[?B]!B`'. Around 25% of the time, *Arcnel* replaces '`K`' with '`A`'. Change the first line of the L-system to `PlantD3 {~3` and process the text. This changes the structure of the plant. Save the L-system. The '`3`' after the '`~`' is the seed that *Arcnel* uses to initialize the pseudorandom number generator before *Arcnel* applies substitution rules. Select *Debug*. Find the

command '\??20' in the *Length* window. Click *Next* until the first '\??20' is selected. To the nearest hundredth, what is the angle of the turtle?

9) The command '\??20' tells the turtle to "randomly" pick an angle from 0° up to (but not including) 20°. Then the turtle "randomly" decides whether to turn clockwise or counterclockwise. Click *Next*. To the nearest hundredth, what is the angle of the turtle?

10) Click *Exit*. Change the order to 30. Select *Debug*. According to the title bar of the *Length* window, what is the length of the command string?

11) Click *Exit*. Change the axiom to '\90DK'. Since '?24' will no longer be in the command string, the turtle will not set the seed to 24. The turtle uses the initial seed which is an unpredictable number that comes from the state of the computer processor. Process the text. The plant should change. Select *Redraw L-system* in the *Edit* menu (or press R). The plant should change again. Change the first line of the L-system to `PlantDS {~` and process the text (click *Yes* in the dialog box that appears). Save the L-system. Since the initial seed used for substitution rule choices is no longer specified, *Arcnel* picks a seed based upon the state of the computer processor. Press R. Do it again at least 3 more times. Select *Process Shown Text* (or press P). Do it at least 3 more times. When *Redraw L-system* is selected, the seed for substitution rule choices doesn't change. It does change when *Process Shown Text* is selected. Press R at least 5 times. Press P at least 5 times. Does pressing R or P tend to change the plant the most?

12) If $\phi = \frac{1+\sqrt{5}}{2}$, then $(360°)(1 - \frac{1}{\phi}) = 137.507764...°$ (the golden angle). Change the order to 4. In the *L-system* window, type the `Florets` L-system. Process the text. Four circles should appear. Save the L-system. Select *Debug* in the *Edit* menu. The title bar of the *Length* window should show a length of 93 (If not, click *Exit* and fix the L-system. Process the text and save the L-system. Then select *Debug* again). Click *Next*. A '[' should be selected in the *Length* window. The next character should be ''' (apostrophe or single quote mark). As *Arcnel* processes the command string of an order to make the next order, the ''' tells *Arcnel* to ignore the next symbols until there is a number. The number is increased by 1. Look at the command string in the *Length* window and see each number after each 'Q'. Write those four numbers.

```
Florets {~ ;Don't confuse `
  Axiom K  ;with the ' below
  K=K\137.5['@Q1M@I@@.7(360]
}
```

13) Click *Next* twice. 'M' should be selected. This tells the turtle to move without drawing. Click *Next*. '@I@' should be selected. This tells the turtle to multiply its crawl length by the inverse of the most recent multiplier. Since the most recent multiplier is 1, the inverse will be 1, and the crawl length won't change. Click *Next* twice. '(360' should be selected. This tells the turtle to draw a 360° arc while turning to the left. Thus a circle will be drawn. Click *Next* until '@Q2' is selected. Click *Next*. To four decimal places, what is the multiplier shown in the *Debug* window? Click *Next* twice. To four decimal places, what is the multiplier?

14) Click *Exit*. Change the order to 400. Certain flowers have a design similar to what is shown. To mark spirals in the design, type the `FloretSpirals` L-system in the *L-system* window. Process the text. Two spirals should be marked with yellow circles, and two should be marked with orange circles. Save the L-system. Study the orange spirals. They are the same type of spiral. Count the total number of spirals of this type. There should be 34. The yellow spirals are a different type of spiral. How many spirals of that type are in the design?

```
FloretSpirals {~ ;order 400, 1114, or 1828
  Axiom [\PK]C5[B][\P:20&B]C4[V]/P:1&V
  P=137.507764
  K=K\P['@Q1M@I@@.7(360]
  B=:19&A ;137.507764*21-8*360=7.663044
  A=B\7.663044['@Q20M@I@@.7(360]
  V=:32&U ;137.507764*34-13*360=-4.736024
  U=V/4.736024['@Q8M@I@@.7(360]
}
```

15) The *golden angle* is also called the *Fibonacci angle*. Select *FloretsCR* in the *Extra* menu. Do it again and again and again (and more if desired). Press L. Find the lines `B=,1,49` and `B=<1,` in the *L-system* window. These lines form the 'B' substitution rule. The first line shows that there are two options. The second line shows one option followed by a comma with nothing after it. This indicates that the second option is to delete 'B' from the command string. According to the comments in the *L-system* window, how often is this done?

16) Press F1. Find the word *stochastic* in the "Randomness" section. What is a stochastic L-system? For more stochastic experience, select *StochasticDendriteC* in the *Extra* menu and then select *Seed Loop* in the *Edit* menu and then click *OK*.

Name _____

L-Systems 11

In arc L-systems, '"' (double quote mark) tells the turtle to remember its location. If a location has been remembered, then '"' tells the turtle to draw a line segment from the remembered location to the current location. This location becomes the remembered location. Don't confuse '"' and ''' (two apostrophes). Start *Arcnel* and change the width to 3. Select *New L-system* in the *Edit* menu. Type the `Diagonals` L-system in the *L-system* window. Process the text. Change the order to 5. *Arcnel* should display a pentagon with its diagonals. Save the L-system. Select *Debug* in the *Edit* menu. The title bar of the *Length* window should show a length of 27. In that window, '"' should be selected. This tells the turtle to remember its current location. *Remember that location*. Click *Next* until the next '"' is selected. The remembered location should be marked. A line segment should be drawn between the remembered location and the current location which will now be the new remembered location. Point the mouse at the turtle and wait. A window should appear. Its second line shows the turtle's coordinates in the *xy*-plane. The third line shows the turtle's location in the *Arcnel* window. In this activity, the word *coordinates* refers to the coordinates in the *xy*-plane.

```
Diagonals {~
  Angle 5
  Axiom "A
  A=F+F+"A
}
```

```
LeafCordate {~
  Angle 40
  Axiom /90=I[A]![A]
  A=[X".]+A
  X=GX
}
```

```
SuspensionBridge {~
  Angle 4
  Axiom @40"[X]|!X
  X=XF[@=1+FYYYZ".]
  Y=FY
  Z=YYZ
}
```

1) Write the turtle's coordinates (the ordered pair shown after "(x, y)=").

2) Click *Next* until the next '"' is selected. Write the turtle's coordinates.

3) Click *Exit*. Type the `LeafCordate` L-system in the *L-system* window. This L-system is derived from an algorithm on p. 123 of *The Algorithmic Beauty of Plants*. Process the text. *Arcnel* should display a tall and thin octagon. Save the L-system. Change the order to 10. Select *Debug*. The text '`/90`' should be selected in the *Length* window. This tells the turtle to turn clockwise 90°. Click *Next*. The first triangle in the *Debug* window shows the direction the turtle would crawl for an '`F`' or '`G`' command. The second triangle shows the direction for a '`D`' or '`M`' command. Point the mouse at each triangle and write each angle.

4) The text '`=I`' should be selected. This tells the turtle to convert a '`DM`' angle to an '`FG`' angle. This command does the *inverse* of the '=' command which tells the turtle to convert an '`FG`' angle to a '`DM`' angle. An '=' used this way should not be confused with the '=' that is the second character in a substitution rule. Click *Next*. Point the mouse at each triangle and write each angle.

5) Click *Next* until '".' is selected. This tells the turtle to store its memory of its current location at the top of the stack. This way, that location will be remembered after a '`]`' command. Then when the turtle comes to a '`[`' command, that memory will be pushed back onto the stack. Remember the location of the turtle and then click *Next* until the next '".' is selected. The turtle looks at the top of the stack to see if a remembered location is stored there. Since one has been, the turtle draws a line segment from that location to its current location. Write the turtle's coordinates.

6) Click *Exit*. Change the order to 18 to draw an outline of a cordate leaf. Select *Toggle Axes* in the *Edit* menu. Then select *Debug*. Click *Next* until '".' is selected. Write the turtle's coordinates.

7) Click *Exit*. Change the name to `LeafCordateSmooth`. Change the angle from '`40`' to '`360`' and process the text. Change the order to 154. Save the L-system. Find *cordate* in a dictionary and write its definition.

8) Change the order to 10. Type the `SuspensionBridge` L-system in the *L-system* window. Process the text. The diagram resembles a suspension bridge. The curve along the top approximates the parabola $y=\left(\frac{x}{40}\right)^2$. Save the L-system. Select *Toggle Axes*. Then select *Debug*. The title bar of the *Length* window should show a length of 1,220. Click *Next* once. The command '"' tells the turtle to remember its current location. Click *Next* once. The command '`[`' places a copy of that memory on the stack. Click *Next* twice. The command '`[`' places another copy of that memory on the stack. Click *Next* once. The command '`@=1`' should be selected. This is an *absolute command* because of the '='. Normally, '`@`' multiplies the crawl length by the value in the command. However, '`@=1`' sets the crawl length to 1 instead of multiplying it by 1. While watching the crawl length in the *Debug* window, click *Next* once. Then click *Next* until '".' is selected. Write the turtle's coordinates.

9) Click *Next* until the next '"." is selected. Write the turtle's coordinates.

10) Click *Next* until the next '"." is selected. Write the turtle's coordinates.

11) Click *Exit*. Change the width to 5. In the *L-system* window, type the `FibonacciArcs` L-system. Process the text. A spiral should appear. Save the L-system. Change the order to 20. Select *Debug*. The title bar of the *Length* window should show a length of 81. An 'L' should be selected. According to the *Debug* window, what is the crawl length?

```
FibonacciArcs {~
  Angle 4
  Axiom :2:1&~$1$&~L:4:3&~$1$&~K
  K=@=*2H:5:4&~B*1$*2$&~:1&K
  B='' ;Don't confuse '' and "
  H=L
  '=
}
```

12) Click *Next* until the next 'L' is selected. What is the crawl length?

13) Click *Next* until the next 'L' is selected. What is the crawl length?

14) The text '@=' occurs ten times in the command string. After each '@=' is a number. Write each of those numbers in the order they occur.

15) Click *Exit*. Change the order to 1. Select *Debug*. Find '*1' and '*2' in the command string. As *Arcnel* applies substitution rules to make order 2, *Arcnel* replaces '*1' with what is between the 1st and 2nd dollar signs. *Arcnel* replaces '*2' with what is between the 3rd and 4th dollar signs. Write what is between the 1st and 2nd dollar signs. Also, write what is between the 3rd and 4th dollar signs.

16) Click *Exit*. Change the order to 2. Select *Debug*. Find ''' in the command string. If *Arcnel* sees a single ''' in a command string as substitution rules are applied, then *Arcnel* searches for a number after the '''. Once a number is found, it is increased by 1. If *Arcnel* sees ''' in a command string as substitution rules are applied, then *Arcnel* reads the number that is immediately after the ''' and remembers that number. *Arcnel* then searches for the next number in the command string and adds the remembered number to that number. Write the number that is immediately after '''. Also, write the next number in the command string.

17) Click *Exit*. Change the order to 17. Select *Debug*. Write what is between the 1st and 2nd dollar signs. Also, write what is between the 3rd and 4th dollar signs.

18) Click *Exit*. Change the order to 18. Select *Debug*. Write the number that is immediately after '''. Also, write the next number in the command string. Then add those numbers and write the sum.

For better understanding of `FibonacciArcs`, observe ''' in a command string. Then observe that for the next order, the first ''' is deleted by the substitution rule ''='. However, the second ''' is not deleted. The reason is that when *Arcnel* sees ''', then *Arcnel* does not apply substitution rules from the character after that ''' up through the number that is changed by the command. Below are sequences from command strings from different L-systems. For the command strings on the left, the L-system has the substitution rule ''='. That substitution rule is not in the L-system for the command strings on the right.

	Delete ''' with rule ''='	Delete ''' with command ':'
Order 0	''3$4$	:''3$4$
Order 1	'3$7$	'4$4$
Order 2	4$7$	'5$4$

The rule ''=' *doesn't* cause the first ''' to be ignored. The first ''' is deleted but not ignored. The command ':' *does* cause the symbol immediately after it to be ignored.

These activities have *not* introduced everything there is to know about the symbols that can be used in L-systems. To develop a better understanding of the meanings of symbols, select *Help* in the *Extra* menu and read about the symbols. Also, study L-systems in "Arcnel.AL" and L-systems in the *Extra* menu. To improve understanding, look at the command strings for orders 1, 2, 3, etc. Even if there is nothing to draw for an order, the command string can be displayed by selecting *Show Command String Text* in the *Edit* menu. Once a command string is displayed, it can be selected by using a mouse and then copied by pressing Ctrl+C. Then it can be pasted into an editor. It is helpful to paste command strings from successive orders into an editor so that the results of substitution rules are more easily seen.