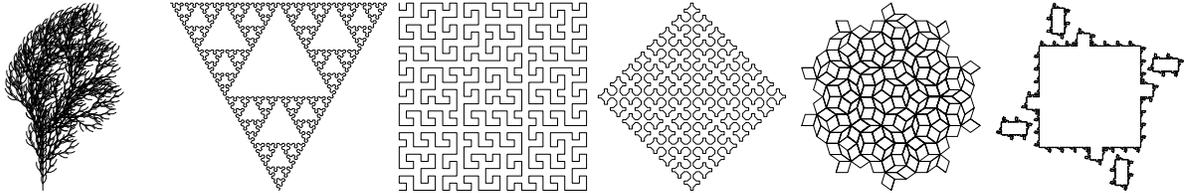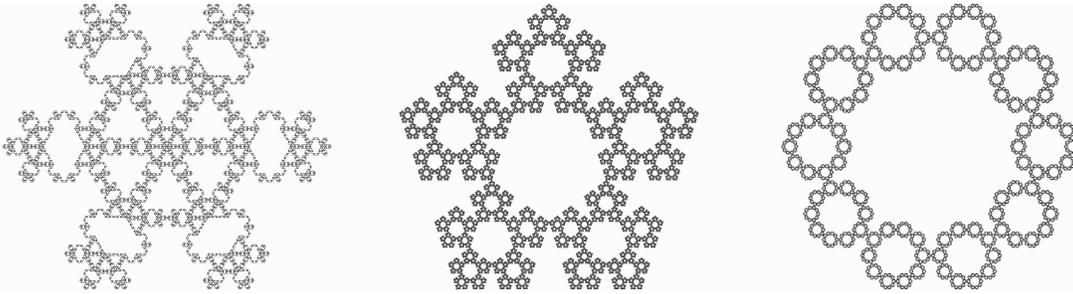# L-Systems 1

These L-system activities are designed to help you create designs like the ones shown below.

Pretend that on the computer screen is a turtle facing towards the right. If you give the turtle the command 'F', the turtle will crawl forward one unit and will leave a mark on the screen as it crawls. If you give the turtle the command 'G', the turtle will crawl forward one unit, but will not leave a mark as it crawls.

So the command 'FGFF' will tell the turtle to crawl forward *one* unit while leaving a mark. Then it will crawl forward *one* unit without leaving a mark. Then it will crawl forward *two* units while leaving a mark. So the resulting drawing will look like this: —— ———

Normally, to make a drawing that will be interesting, we want the turtle to be able to turn to the left or the right. The command '+' makes the turtle turn to its left (i.e., counterclockwise). The command '–' makes the turtle turn to its right (i.e., clockwise). Before giving the turtle the commands '+' or '–', we will tell the turtle how far to turn. The instruction 'Angle 4' will tell the turtle to turn $\frac{1}{4}$ of a circle (i.e., 90°) whenever it is given a command of '+' or '–'. The instruction 'Angle 6' will tell the turtle to turn 60° whenever we command it to turn (since 360°÷6 = 60°). Suppose the turtle is told 'Angle 12'. Then it is commanded 'F+F+F----F+F'. It will 1st crawl forward one unit. Then it will turn counterclockwise 30° (since 360°÷12 = 30°). Then it will crawl forward another unit (so the drawing now looks like this: ——). Then it will turn left 30° again and crawl forward (now the drawing looks like this: ___/). Then it will turn right (i.e., clockwise) 4 times (for a total of 120°). Then it will crawl forward once (the drawing now looks like this: ___/\). Then it will turn counterclockwise and crawl forward to finish the drawing.

Try to understand how the commands 'F+F+F----F+F' produce this drawing: ___/___.

To learn how to instruct the computer to make designs, start the program *Elsys*. Click *Edit* on the menu bar and then click *New L-system*.

After reading this paragraph, edit the text in the *L-system* window that appears so that it contains the commands that follow this paragraph. The first line of a formula contains the name of the formula and a brace ('{'). The brace tells the computer that the formula is now beginning. The next line tells the turtle how far to turn when it receives a '+' or '–' command. You do not yet need to understand the significance of the word 'Axiom'. Just make sure it is in the L-system. The commands for the turtle come after the word 'Axiom'. The formula ends with a brace ('}'). To quickly change the name of the L-system, double-click 'New' in the *L-system* window and type 'Sample1 ' (note the space at the end). Edit the text in the *L-system* window until it contains these commands:

```
Sample1 {
  Angle 12
  Axiom F+F+F----F+F
}
```

Be sure that 'Sample1' has no space in it. Put a space *after* 'Sample1' and *before* the brace ('{').

Save this paper for future reference.

To have the computer process the `Sample1` L-system, click *Edit* on the menu bar and then click *Process Shown Text*. The *Elsys* window should then draw this design: ____ .

To save the `Sample1` L-system, click *File* on the menu and then click *Save L-system To File*. Then type your initials followed by ".AL". For example, if you initials are PQR, then type PQR.AL for the file name. Then click the *Save* button.

Now select *New L-system* in the *Edit* menu and edit the text in the *L-system* window that appears so that it contains the commands that follow this paragraph. You do not need to type the text that is after a semicolon (';'). But you do need to type all of the text that is before a semicolon. If a line does not contain a semicolon, then type everything on that line.

```
Pentagon { ; Computer ignores text after a semicolon.
  Angle 5  ; 360/5 = 72. The text after a semicolon doesn't need to be typed.
  Axiom F+F+F+F+F ; '+' = 'turn counter-clockwise one-fifth of a circle'
}
```

To have the computer process the `Pentagon` L-system, select *Process Shown Text* in the *Edit* menu. The *Elsys* window should then draw a pentagon. To save the `Pentagon` L-system, select *Save L-system To File* in the *File* menu. Then select the ".AL" file you saved above and click the *Save* button.

Now select *New L-system* in the *Edit* menu and edit the text in the *L-system* window that appears so that it contains the following commands:

```
Star { ; If text after a ';' isn't typed, then the ';' doesn't need to be typed either.
  Angle 10 ; 360/10 = 36. '-' = 'turn clockwise one-tenth of a circle'
  Axiom F----F----F----F----F
}
```

Select *Process Shown Text* in the *Edit* menu. The *Elsys* window should then draw a 5-pointed star. Using the procedure taught above, now save the `Star` L-system to your ".AL" file.

To watch how the turtle draws the star, select *Debug* in the *Edit* menu. In the *Debug* window, click *Next* on the menu bar to have the turtle move one unit. Answer the following questions (you may write on this paper):

1) Click *Next* again. The leftmost triangle in the *Debug* window should have rotated. Point the mouse at that triangle. The computer should display an angle measure. What is that angle measure?

2) Click *Next* again. Point the mouse at the leftmost triangle in the *Debug* window. The computer should display an angle measure. What is that angle measure?

3) Click *Next*. Write the new angle measure.

4) Click *Next*. Write the new angle measure.

5) Click *Next*. The turtle should crawl forward one unit. Now click *Next* four times to make the turtle turn four times. Write the new angle measure.

6) Click *Auto*. Watch the turtle. Once the turtle is done moving, what is the angle measure?

7) Click *Next*. This exits *Debug* mode. Select *Open* in the *File* menu. Select the ".AL" file you saved and click the *Open* button. Click `Pentagon` in the list of L-systems. Click *OK*. An *Order* window should appear. A later activity teaches about the "Order" concept. For now, just click *OK* in the *Order* window. This should draw a pentagon. Select *Debug* in the *Edit* menu. A window on the screen should show the commands F+F+F+F+F. The title bar of that window should show the text "Length:" followed by a number. Write that number.

8) Click *Next* in the *Debug* window. The first '+' in the *Length* window should be selected. This means that '+' is the next command. Click *Next* to perform that command. Write the new angle measure.

9) Click *Auto*. Write the angle measure once the turtle stops moving. You may now exit from *Elsys*.

L-Systems 2

Start the program *Elsys*. Click *Edit* on the menu bar and then click *New L-system*. Edit the text in the *L-system* window until it contains these commands:

```
KochCurve2 { ; You do not need to type text that is after a semicolon.
  Angle 6  ; 360/6=60. '-' = 'turn clockwise one-sixth of a circle'
  Axiom FGF  ; Don't forget there are braces at the beginning and the end!
  F=F+F--F+F  ; The lines after the axiom line contain substitution rules.
  G=GGG
}
```

To have the computer process the `KochCurve2` L-system, select *Process Shown Text* in the *Edit* menu. Save the L-system as instructed in the first activity. Then select *L-system Order* in the *Edit* menu. In the window that pops up, type the digit 0 (zero). Click *OK*. Since the formula contains the command '`Axiom FGF`', the turtle draws a segment. Then it moves forward without leaving a mark. Then it draws another segment.

Now select *L-system Order* in the *Edit* menu. In the window that pops up, type the digit 1 (one). Click *OK*. Since the order is 1, the computer uses the commands '`F=F+F--F+F`' and '`G=GGG`' to change the commands for the turtle. The command '`F=F+F--F+F`' tells the computer to replace each '`F`' in the axiom with the text '`F+F--F+F`'. Likewise, the command '`G=GGG`' tells the computer to replace the '`G`' in the axiom with the string '`GGG`'. So the computer replaces the original command '`FGF`' with the command '`F+F--F+FGGGF+F--F+F`'. The turtle uses this command to draw the diagram. The computer automatically resizes the diagram so that it fits on the screen. To watch how the turtle draws this design, select *Debug* in the *Edit* menu.

1) A window on the screen should show the commands F+F--F+FGGGF+F--F+F. The title bar of that window should show the text "Length:" followed by a number. Write that number.

2) Click *Next* in the *Debug* window. Write the symbol that is selected in the *Length* window.

3) Click *Next* in the *Debug* window. Point the mouse at the leftmost triangle in the *Debug* window. What is the angle measure?

Click *Exit* in the *Debug* window. If the order were '2', the computer would perform the substitutions a second time. The text that results at each step is shown in the following diagram. Each time, the computer replaces each '`F`' with '`F+F--F+F`' and replaces each '`G`' with '`GGG`'.
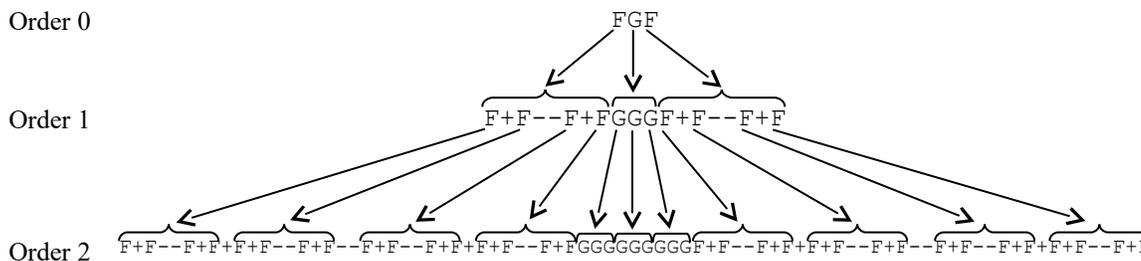
In this example, the substitution '`F=F+F--F+F`' is equivalent to replacing each straight segment with the shape ___/___. Change the order to 2 and see the substitution occur. Then change the order to 3 and see the substitution occur again.
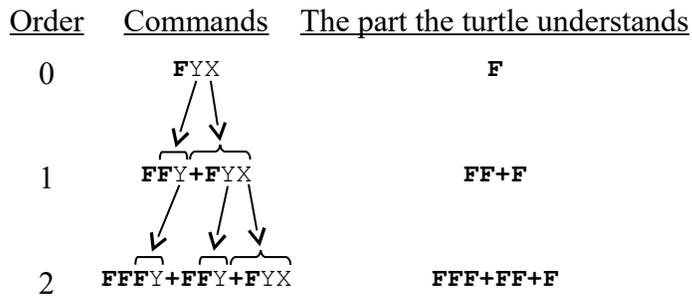
Click *Edit* on the menu bar and then click *New L-system*. Edit the text in the *L-system* window until it contains these commands:

```
TriSpiral {
  Angle 3 ;360/3 = 120. '+' = 'turn counter-clockwise one-third of a circle'
  Axiom FYX ; As the turtle crawls, it ignores symbols it doesn't understand.
  Y=FY   ;When the turtle sees a Y, the turtle ignores it.
  X=+FYX
}
```

This formula contains symbols '`X`' and '`Y`'. The turtle does not understand these symbols, so it ignores them. But as the computer computes orders 1, 2, 3, etc., it will follow the substitution rules '`Y=FY`' and '`X=+FYX`'.

Select *Process Shown Text* in the *Edit* menu. Save the L-system. Select *L-system Order* in the *Edit* menu. Change the order to 0 and click *OK*. Also try orders 1, 2, 3, and 14. Select *Curve Width* in the *Edit* menu. Type the digit 3 and click *OK*.

Order	Commands	The part the turtle understands
0	**F**YX	**F**
1	**FF**Y+**F**YX	**FF+F**
2	**FFF**Y+**FF**Y+**F**YX	**FFF+FF+F**

Sometimes, to increase understanding of what the computer is doing, it can be helpful to have the turtle color the segments it draws. Edit the text in the *L-system* window until it contains these commands:

```
TriSpiralColor { ; In the default color palette, color 15 is white.
  Angle 3      ; Text after a semicolon is ignored by the computer.
  Axiom C15FYX ; 'C15' tells the turtle to chose color 15.
  Y=>1FY       ; '>1' tells the turtle to decrease the color number by 1.
  X=+C15FYX    ; After each turn, chose color 15 again.
}
```

IMPORTANT NOTICE (**Do not discard**)
If you need to change an L-system after saving it, open the ".AL" file in *Notepad* and edit that file and then save it. Then select *Open* in the *File* menu in *Elsys* to reopen the changed ".AL" file.

Select *Process Shown Text* in the *Edit* menu. Save the L-system. Try orders 1, 2, 3, and 4 to see how the turtle colors different segments. Then try order 14. Notice how the first segment at the bottom-left is white (color 15). The next segment is color 14. The next segment is color 13, etc. Then when the turtle turns 120°, it chooses color 15 (white) again. To see the color palette, select *Show Editable Color Map* in the *Edit* menu.

4) Point the mouse at the white square in the color palette. The computer should show two lines of information. The first line shows that the color number is 15. Write the entire second line of information.

5) The second line of information shows the red, green, and blue values for the color. Move the mouse around until you find color # 14. What are the red, green, and blue values for color 14?

Change the order to 3. For *each one* of the following *three* L-systems, first edit the *L-system* window so that it contains the L-system. Then select *Process Shown Text* in the *Edit* menu. Then save the L-system.

```
Snow {
  Angle 6
  Axiom F+F+F+F+F+F
  F=F+FF+F--F--F+FF+F
}
```
```
SnowColor { ; Try order 4 or 5
  Angle 6 ; '<2' --> increase color # by 2
  Axiom C2F+<2F+<1F+<2F+<4F+<2F
  F=F+FF+F--F--F+FF+F
}
```
```
SnowColor2 { ; Try order 1 or 2
  Angle 6 ; '>2' --> decrease color # by 2
  Axiom F+F+F+F+F+F
  F=C13F+>2F>2F+>2F-->1F-->1F+>1F>1F+>1F
}
```

The Snow design should look similar to the top-left design on the *L-Systems 1* activity. If you saved a flawed L-system, see the IMPORTANT NOTICE above.

6) Select *Open* in the *File* menu. Select the ".AL" file you saved. Choose Snow in the L-system list and click *OK*. Type 0 for the order and click *OK*. Name the shape produced.

7) Change the order to 1. Choose *Show Command String Text* in the *Edit* menu. The title bar of the *Length* window shows the length of the command string text. What is that length?

8) Select *Choose L-system* in the *File* menu. Choose SnowColor2 in the L-system list and click *OK*. Make sure the order is 1 and click *OK*. Select *Show Command String Text* in the *Edit* menu. What is the length displayed in the title bar of the *Length* window?

9) Select *Choose L-system* in the *File* menu. Choose SnowColor in the L-system list and click *OK*. Make sure the order is 1 and click *OK*. Select *Debug* in the *Edit* menu. What is the length displayed in the title bar of the *Length* window?

10) Click *Next* in the *Debug* window. Remember the new color of the turtle. Now continue to click *Next* until the turtle turns a different color. What is the measure of the angle for the turtle?

11) Click *Exit* in the *Debug* window. Change the order to 4. Choose *Show Command String Text* in the *Edit* menu. A window should pop up. Don't click any buttons. According to the window that pops up, how long is the command string?

12) Click *No*. Change the order to 3. Choose *Show Command String Text* in the *Edit* menu. How long is the command string?

L-Systems 3

Start *Elsys*. Select *New L-system* in the *Edit* menu. For *each* of the following three L-systems, type the L-system in the *L-system* window and then process the text and then save the L-system.

```
KochCurve3 {
  Angle 6
  Axiom F!F!F
  F=F+F--F+F
}
```

```
KochCurve3Color {
  Angle 6
  Axiom C5FC7!F!C11F
  F=F+F--F+F
}
```

```
Zigzag {
  Angle 3
  Axiom F
  F=F!-F
}
```

The '!' command tells the turtle to **reverse** the meaning of '+' and '-'. In the axiom 'F!F!F' the '!' command has no effect (since there are no '+' or '-' commands). But when the computer applies the substitution rule, the '!' command does have effect. After applying the substitution 'F=F+F--F+F' to the axiom 'F!F!F', the command will be 'F+F--F+F!F+F--F+F!F+F--F+F'. Normally, '+' means '**turn counterclockwise**' and '-' means '**turn clockwise**'. After the turtle receives the 1st '!' command, it will interpret '+' as 'turn clockwise' and '-' as 'turn counterclockwise'. When the turtle is given the next '!' command, it again reverses the meanings of '+' and '-'. So now they are back to their original meanings.

Open the file to which you saved the above three formulas and choose KochCurve3 and click *OK*. Type the digit 1 for the order and click *OK*. Then select *Debug* in the *Edit* menu. Notice that an 'N' is displayed in the *Debug* window. This means that '+' and '-' have their **normal** meaning. Click *Next* in the *Debug* window until the first '!' is selected in the *Length* window. The *Debug* window should still show an 'N'.

1) Click *Next* one more time. Write the letter that is now displayed instead of the 'N'.

2) Click *Next* two more times. What is the angle measure for the turtle?

3) Click *Next* in the *Debug* window until the second '!' is selected in the *Length* window. Then click *Next* three more times. What is the angle measure for the turtle?

4) Click *Exit*. Select *Choose L-system* in the *File* menu. Choose KochCurve3Color, click *OK*, type the digit 2 for the order, and click *OK*. Select *Debug* in the *Edit* menu. Observe the 'N' in the *Debug* window. Click *Auto* in the *Debug* window and watch both the 'N' and the turtle. What is the color of the design when the meanings of '+' and '-' are reversed?

5) Click *Exit*. Select *Choose L-system* in the *File* menu. Choose Zigzag, click *OK*, type the digit 4 for the order, and click *OK*. Select *Debug* in the *Edit* menu. Click *Auto*. When the turtle is finished moving, what is the meaning of '-'? (Use *words* in the answer. Read the top paragraphs on this page if you have questions.)

Click *Exit*. Enter the following L-system, process it, and save it.

```
OctSpiralColor {
  Angle 8
  Axiom C16F
  F=F<1+@.99F ;'@' is the 'at' symbol which is found on the key for '2' on many keyboards
}
```

The '@.99' command tells the turtle to crawl 99/100 as far as it used to crawl when it would be commanded 'F' or 'G'. For order 2 (i.e., after 2 substitutions), the command will be 'C16F<1+@.99F<1+@.99F<1+@.99F'. For the 1st 'F', the turtle will crawl the normal distance. For the 2nd 'F', the turtle will crawl 0.99 of the original distance. For the 3rd 'F', the turtle will crawl 0.99 of that new distance which means that it will crawl 0.9801 of the original distance.

6) Do order 8 of OctSpiralColor. Select *Spectrum Map* in the *Edit* menu. Select *Debug* in the *Edit* menu. Observe the number 1 displayed in the *Debug* window. Click *Next* until that number changes. Write the new number.

   Hint: Instead of clicking *Next* in the *Debug* window, you may press $\boxed{\text{N}}$ on the keyboard if that window is active. Also, you may press $\boxed{\text{A}}$ instead of clicking *Auto*, and you may press $\boxed{\text{Esc}}$ instead of clicking *Exit*.

7) Click *Next* until the number changes again. Write the new number.

8) Click *Next* until the number changes again. Write the new number.

Click *Exit*. **Change the order to 3**. Select *Elsys Map (Default)* in the *Edit* menu. Select *New L-system* in the *Edit* menu. For *each* of the following three L-systems, type the L-system in the *L-system* window and then process the text and then save the L-system. The '|' symbol may look like '¦' on the keyboard.

```
KochAlt2 {              KochAlt6 {                    KochAlt6Color {
  Angle 6                 Angle 6                       Angle 6
  Axiom F|F               Axiom F|FF|F+F|FF|F+F|FF|F     Axiom C2F|F<2F|F+<1F|F<2F|F+<2F|F<2F|F
  F=F+F--F+F              F=F+F--F+F                     F=F+F--F+F
}                       }                             }
```

If the 'Angle' command is an even number, then the '|' command tells the turtle to turn 180°. So in KochAlt2, the axiom 'F|F' is equivalent to 'F+++F'. At order 0, 'F|F' causes the turtle to crawl forward, then turn around and crawl back the way it came. So at order 0, the extra 'F' is not visible. At order 1 (after substituting 'F+F--F+F' for each 'F' in 'F|F'), the command will be 'F+F--F+F|F+F--F+F'.

9) Open the file to which you saved the above 3 formulas. Select KochAlt2 and let the order be 1. How many line segments are visible in the design?

10) Change the order to 2. The design should have 3 polygonal regions. How many sides does the central polygon have?

11) Change the order to 3. How many polygonal regions are in the design?

12) Select *Choose L-system* in the *File* menu. Choose KochAlt6 and let the order be 1. Select *Debug* in the *Edit* menu. Click *Auto*. When the turtle is finished moving, what is its angle measure?

13) Click *Exit*. Change the order to 5. Select *Show Command String Text* in the *Edit* menu. What is the length of the text?

14) Select *Choose L-system* in the *File* menu. Choose KochAlt6Color and let the order be 2. Select *Debug* in the *Edit* menu. Click *Next* until the turtle is at the rightmost point in the design. Click *Next* one more time. The turtle should have changed its direction. By how many degrees did it change its direction?

15) Keep clicking *Next* until the turtle changes its color. What is its new color?

16) Keep clicking *Next* until the turtle changes its color again. What is its new color? (Make sure that the answers to #15 and #16 are different. If they are the same, you probably need to change the answer for #15)

17) Click *Exit*. Press F1 on the keyboard. The *Elsys Help* window should appear. According to the information in it, who introduced L-systems?

L-systems 4

Start *Elsys*. Select *New L-system* in the *Edit* menu. For *each* of the following three L-systems, type the L-system in the *L-system* window and then process the text and then save the L-system.

```
DoubleSpiral {
  Angle 40
  Axiom ----[F]|F
  F=F+@.98F
}
```

```
FlakeV {
  Angle 5
  Axiom F+F+F+F+F
  F=F+F[+F]--F+F
}
```

```
FlakeVColor {
  Angle 5
  Axiom C2F+<2F+<1F+<2F+<4F
  F=F+F[+F]--F+F
}
```

The '[' command tells the turtle to remember its current condition (its position, direction, color, length of crawl, and meaning of '+' and '-'). Then when the turtle is given the ']' command, it goes back (without leaving any marks) to the condition it had been in when it was given the '[' command.

1) Open the file to which you saved the above formulas. Choose `DoubleSpiral` and let the order be 5. Choose *Debug* in the *Edit* menu. Click *Next* until the '[' is selected in the *Length* window. Then click *Next* again. The *Debug* window should now show two lines of information. Look at the two triangles in the left column. What is the angle of the top triangle?

2) What is the angle of the bottom triangle?

3) Click *Next* until the ']' is selected in the *Length* window. Now what is the angle of the top triangle?

4) What is the angle of the bottom triangle?

5) The right column in the *Debug* window shows the crawl length for the turtle. Write both the current crawl length (top line) and the remembered crawl length (bottom line) with as much precision as is displayed in the *Debug* window.

6) While watching the turtle, click *Next*. The remembered conditions have now become the current conditions. Click *Next* again. What is the current angle measure for the turtle?

7) Click *Next* two more times. By how many degrees did the angle measure change?

8) Click *Exit*. Select *Choose L-system* in the *File* menu. Choose `FlakeV` with an order of 1. How many pentagons are in the design?

9) Change the order to 5. Select *Show Command String Text* in the *Edit* menu. A window should pop up. Carefully read it, but don't click any buttons. According to that window, what is the length of the command string?

10) Click *No*. Select *Choose L-system* in the *File* menu. Choose `FlakeVColor` with an order of 2. Choose *Debug* in the *Edit* menu. Click *Next* until the first '[' is selected in the *Length* window. Then click *Next* until the first ']' is selected. Then click *Next* until the 2nd '[' is selected. Then click *Next* until the 2nd ']' is selected. Then click *Next* until the 3rd '[' is selected. Then click *Next* until the 4th '[' is selected. Click *Next* three more times. The *Debug* window should show 3 lines of information. The left triangle in the first line shows the current angle of the turtle. The left triangle of the 2nd line shows the most recently remembered angle. The left triangle of the 3rd line shows an angle that had been remembered earlier. Write all three angle measures, starting with the current angle.

11) The *Elsys* window shows two small circles drawn on the design. The one circle is larger than the other. The smaller circle shows the most recently remembered location. The larger circle shows an earlier remembered location. Carefully watch the larger circle as you click *Next*. What happens to that circle?

12) Click *Next* until the turtle turns green. What is its angle measure?

13) Click *Exit*. Change the order to 5. Observe how the different colors are distributed around the design. Studying colors can help to increase understanding regarding how the turtle crawls. Press F1 . Find where *Elsys Help* teaches that the '[' command is called a "Push" command. What is the ']' command called?

L-Systems 5

Start *Elsys*. Select *New L-system* in the *Edit* menu. For <u>each</u> of the following two L-systems, type the L-system in the *L-system* window and then process the text and then save the L-system.

```
KochSnow {
  Angle 12
  Axiom +F----F----F
  F=F++F----F++F
}
```

```
KochSnowBuild {
  Angle 12
  Axiom +FY----FY----FY
  F=    ;This is not a mistake
  X=FY
  Y=FX[F]++FX----FX++FX
}
```

The Koch snowflake is named after Helge von Koch. *KochSnow* can help you visualize the design.

1) Open the file to which you saved the above formulas. Choose `KochSnow` and let the order be 0. How many line segments are in the design?

2) Change the order to 1. How many line segments are in the design?

3) Change the order to 2. How many line segments are in the design?

4) Change the order to 5. This should give you some idea of how the Koch snowflake would look. An infinite order would be needed to produce the actual Koch snowflake. Select *Show Command String Text* in the *Edit* menu. What is the length of that text?

5) Select *Choose L-system* in the *File* menu. Choose `KochSnowBuild` and let the order be 0. This L-system contains the substitution rule '`F=`' (with nothing after '='). This rule will replace '`F`' with nothing (i.e., it will delete each '`F`'). This L-system shows one way of visualizing the construction of the Koch snowflake. What special type of triangle is displayed when the order is 0?

6) Change the order to 1. This places an equilateral triangle on the middle third of each side of the original triangle. Now change the order to 2. This should erase the boundaries between the original triangle and the little triangles. Now change the order to 3. How many little triangles does this add to the design?

7) Change the order to 4. This should erase the boundaries between the main design and the little triangles. Change the order to 5. Select *Debug* in the *Edit* menu. Click *Next* until the first '`]`' is selected in the *Length* window. Write both the current angle measure for the turtle and the remembered angle measure.

8) Click *Next* three times. What is the current angle measure?

Click *Exit*. Select *New L-system* in the *Edit* menu. For <u>each</u> of the following two L-systems, type the L-system in the *L-system* window and then process the text and then save the L-system.

```
QBlock2 {
  Angle 8
  Axiom F++F++F++F
  F=F[++F]G[@Q8+F]G++G|F++F
  G=GGGG ;4 G's
}
```

```
Quilt2 {
  Angle 8
  Axiom F++F++F++F
  F=F[++F--F]@I2G@Q2+F++F++F++F@IQ2+G@2[++!F]F
  G=GGG ;3 G's
}
```

9) In `QBlock2`, the command '`@Q8`' multiplies the turtle's crawl length by the square root of 8. In `Quilt2`, the command '`@I2`' multiples the crawl length by the inverse of 2 (i.e., $\frac{1}{2}$). The command '`@IQ2`' multiplies the crawl length by the inverse of the square root of 2. Do order 1 of `Quilt2`. How many squares are tilted?

10) The commands for drawing tilted squares are found between the commands '`@Q2`' and '`@IQ2`'. The command '`@IQ2`' restores the crawl length to what it had been before the '`@Q2`' command. Select *Debug* in the *Edit* menu. Click *Next* until the first '`@I2`' is selected. What is the current crawl length?

11) Click *Next* once. What is the new crawl length?

12) Click *Next* twice. Write the new crawl length rounded to three decimal places.

13) Click *Next* until the first '@2' is selected. What is the current crawl length?

14) Click *Next* once. What is the new crawl length?

15) Click *Exit*. Change the order to 5. Select *Show Command String Text* in the *Edit* menu. A window should pop up. Do not click any buttons. According to the window that pops up, what is the length of the command string?

16) Click *No*. Open the file to which you saved the formulas. Choose QBlock2 and let the order be 1. Select *Show L-system Text* in the *Edit* menu. The command for drawing one side of the large tilted square is the 'F' in '[@Q8+F]'. Notice that at the ']' command, the turtle returns to the condition it had been in when it received the '[' command – including the size of the crawl. So the command '@IQ8' was not needed to restore the crawl length to its original size. How many squares are in the design?

17) Select *Debug* in the *Edit* menu. Click *Next* until the first '@Q8' is selected. What is the crawl length?

18) Click *Next* once. Write the current crawl length rounded to 3 decimal places.

19) Click *Next* twice. What is the current angle measure?

20) Click *Next* once. Write both the current angle and crawl length.

21) Click *Exit*. Change the order to 2. How many tilted squares are in the design?

22) Study the central square in the order 2 diagram. Then study the central squares for orders 3, 4, and 5. How does the central square in an odd order compare to the central square in an even order?

L-Systems 6
Start *Elsys*. Select *New L-system* in the *Edit* menu. For <u>each</u> of the following two L-systems, type the L-system in the *L-system* window and then process the text and then save the L-system.

```
FlakeIXb {
  Angle 9
  Axiom F+F+F+F+F+F+F+F+F
  F=F+F+F[+F+F+F]|F+F+F
;For Angle 9, '|'='+++++' for default meaning of '+'
}
```

```
MultiSpiral6G {; Try order 88
  Angle 40
  Axiom ----[FX]|FX
  X=[@.35Y][|@.35Y]+@.98F+@.98FX
  Y=++@.964FY
}
```

1) If the # in the command 'Angle #' is even, then '|' tells the turtle to turn 180°. If the # is odd, then the turtle will turn counterclockwise 180° and will keep turning until it reaches a multiple of 360° ÷ #. For 'Angle 9', 360° ÷ 9 = 40°. Since 4(40°) = 160° and 5(40°) = 200°, '|' tells the turtle to turn counterclockwise 200° (equivalent to turning clockwise 160°). So when '+' and '-' have their normal meanings, the '|' is equivalent to '+++++' which is equivalent to '----'. Open the file to which you saved the above formulas. Choose FlakeIXb and let the order be 1. Select *Debug* in the *Edit* menu. Click *Next* until the first '|' is selected in the *Length* window. What is the current angle measure for the turtle?

2) Click *Next* once. What is the current angle measure for the turtle?

3) Click *Next* until the next '|' is selected in the *Length* window. What is the current angle measure for the turtle?

4) Click *Next* once. By how many degrees did the angle measure change?

5) Click *Exit*. MultiSpiral6G was derived from the DoubleSpiral formula of a previous activity. Order 8 works well for DoubleSpiral, but MultiSpiral6G needs a much higher order. Select *Choose L-system* in the *File* menu. Choose MultiSpiral6G with an order of 88. Select *Show Command String Text* in the *Edit* menu. According to the window that pops up, what is the length of the command string?

6) Click *No*. Change the order to 3. Select *Debug* in the *Edit* menu. Click *Next* until the first ']' is selected in the *Length* window. What is the current angle measure for the turtle?

7) Click *Next* once. What is the current angle measure for the turtle?

8) Click *Next* until the next ']' is selected in the *Length* window. What is the current angle measure for the turtle?

9) Click *Exit*. Open "Elsys.AL" (**Important:** If it doesn't exist, select *Write Elsys.AL* in the *File* menu and click the *Save* button. Then open "Elsys.AL"). Choose FlakeX and let the order be 4. Then change the order to 1. The design consists of 10 congruent regular polygons joined together. What is the name for those polygons?

10) Open "Fractint.L" (If it doesn't exist, select *Write Fractint.L* in the *File* menu and click the *Save* button. Then open "Fractint.L"). Choose Bush and let the order be 4. Select *Show L-system Text* in the *Edit* menu. Write the axiom (If you don't know what the axiom is, press F1 and read *Elsys Help* until you understand).

11) Select *Choose L-system* in the *File* menu. Choose Curve4 and let the order be 6. Also try orders 7, 8, and 9. Concerning the overall design, how do the odd and even orders differ from each other?

12) Now choose Fass1 and let the order be 4. Then choose Peano2 and let the order be 4. Select *Show L-system Text* in the *Edit* menu. Write the axiom.

13) Now choose Penrose4 and let the order be 4. Select *Show L-system Text* in the *Edit* menu. Write the axiom.

14) Now choose Island1 and let the order be 2. The design consists of three sizes of islands. There is one large island. How many medium islands are in the design?

If you develop any interesting designs, the instructor would be interested in seeing the formulas and pictures.

L-Systems 7

Observe that each of the following L-systems has a *tilde* (~) immediately after the opening brace. This tilde tells *Elsys* that the L-system is an **arc L-system**. In an arc L-system, 'L' tells the turtle to draw an arc of a circle while turning to the left (i.e., counterclockwise), and 'R' tells the turtle to draw an arc of a circle while turning to the right (i.e., clockwise). Start *Elsys*. Select *New L-system* in the *Edit* menu. For <u>each</u> of the following four L-systems, type the L-system in the *L-system* window and then process the text and then save the L-system.

```
Rapids {~ ;order 4     Spiral4c {~ ;order 5     Spirals8c {~ ;order 50     Spirals9c {~ ;order 70
  Angle 5                Angle 4                  Angle 8                    Angle 9
  Axiom A                Axiom C16L               Axiom C16Y                 Axiom C16KZ
  A=RLA                  L=L@.9<1L                X=@.93LX                   K=@.9LKR@I.9
}                      }                          Y=+@.9[@.4LX]G<2Y          Z=<1@.9LKZ
                                                }                          }
```

1) Open the file to which you saved the above formulas. Choose `Rapids` and let the order be 4. Select *Debug* in the *Edit* menu. What is the length of the command string? How many times does 'L' appear in the command string?

2) Click *Next* once. What is the current angle measure for the turtle? Study the arc traced by the turtle. Did the turtle turn clockwise or counterclockwise as it made the arc? By how many degrees did the turtle turn as it made the arc? Hint: the initial angle of the turtle can be thought of as being 360°.

3) Click *Auto*. What is the final angle measure for the turtle?

4) Click *Exit*. Select *Choose L-system* in the *File* menu. Choose `Spiral4c` with an order of 5. Select *Debug* in the *Edit* menu. What is the length of the command string?

5) Carefully watch the turtle. Click *Next* once. What happened to the turtle when *Next* was clicked?

6) Click *Next* again. Did the turtle turn clockwise or counterclockwise as it made the arc? By how many degrees did the turtle turn?

7) Click *Next* until the third 'L' is selected in the *Length* window. When the turtle draws an arc, it uses its linear crawl length for the radius of the circle that contains the arc it will draw. What is the current crawl length for the turtle?

8) Click *Auto*. When the turtle is done crawling, write the displayed crawl length to 5 decimal places.

9) Click *Exit*. Select *Choose L-system* in the *File* menu. Choose `Spirals8c` with an order of 50. Press Ⓦ on keyboard and change the width to 3. The design should show a spiral of spirals. If we start counting with the biggest spiral at the bottom of the window and go counterclockwise, the 3ʳᵈ spiral should be yellow and the 4ᵗʰ spiral should be green. Which spiral is the next yellow spiral? Which spiral after that one is the next yellow spiral?

10) Select *Debug* in the *Edit* menu. Click *Next* until the first ']' is selected in the *Length* window. Write the current crawl length rounded to 6 decimal places.

11) Click *Next* two more times. What is the current angle measure for the turtle?

12) Click *Exit*. Select *Choose L-system* in the *File* menu. Choose `Spirals9c` with an order of 70. The design should show a spiral of connected spirals. Select *Debug* in the *Edit* menu. What is the length of the command string?

13) Click *Next* three times. What is the current angle measure for the turtle?

14) Look over the command string and find the first '<1'. Click *Next* once. Then press Ⓝ on the keyboard until that first '<1' is selected. What is the current angle measure for the turtle?