Primitive Pythagorean Triples

Name _____________

Do work and write answers on this paper

1. If a, b, and c are natural numbers such that $a^2 + b^2 = c^2$, then $[a\ b\ c]$ is a **Pythagorean triple**. The triples [3 4 5] and [9 12 15] are both Pythagorean triples. A Pythagorean triple is **primitive** if the numbers have *no common factors* (other than 1). The triple [3 4 5] is primitive. The triple [9 12 15] is not primitive. What is the common factor (other than 1) in the triple [9 12 15]?

2. Calculate $\begin{bmatrix} 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -2 \\ 2 & 2 & 3 \end{bmatrix}$. This should result in a primitive Pythagorean triple.

3. To make sure that you did the previous multiplication correctly, check to see if your answer is a *Pythagorean triple*. Then check to make sure that your answer is a *primitive* Pythagorean triple. Now consider the following three matrices:

$$U = \begin{bmatrix} 1 & 2 & 2 \\ -2 & -1 & -2 \\ 2 & 2 & 3 \end{bmatrix} \qquad D = \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix} \qquad A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix}$$

Multiplying a primitive Pythagorean triple by one of these matrices will produce another primitive Pythagorean triple. Let $P = \begin{bmatrix} 3 & 4 & 5 \end{bmatrix}$. **So for #2 you calculated *PU*.**

Calculate $PD = \begin{bmatrix} 3 & 4 & 5 \end{bmatrix} \begin{bmatrix} -1 & -2 & -2 \\ 2 & 1 & 2 \\ 2 & 2 & 3 \end{bmatrix}$ to find another primitive Pythagorean triple.

4. Multiply your answer to #3 by U. The answer should be the Pythagorean triple PDU.

5. Take your answer to #2 and multiply by D. The answer will be PUD.

6. **Be sure your answers to 2-5 are Pythagorean triples**. Notice that $PUD \neq PDU$. Multiplication of matrices is not generally commutative. Multiply the answer to #2 by U to find PUU.