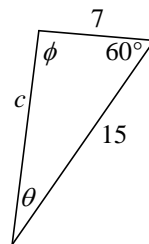
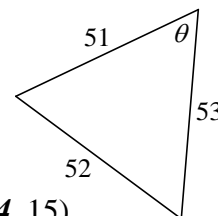
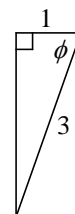
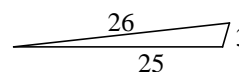
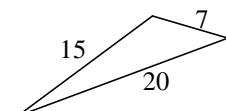
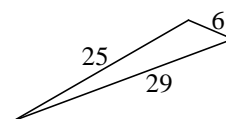


Special Triangles

Do work and write answers on notebook paper



1. Use the law of cosines ($c^2 = a^2 + b^2 - 2ab \cos C$) to find c .
2. Use the law of cosines or the law of sines ($\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$) to find θ (theta) to *two* decimal places.
3. Use subtraction to find ϕ (phi) to *two* decimal places.
4. Consider these formulas: $a = m^2 - n^2$, $b = 2mn - n^2$, $c = m^2 - mn + n^2$. Find (a, b, c) if $(m, n) = (3, 1)$.
5. For the triangle found in the previous problem, use the law of cosines to find C (i.e., the measure of the angle opposite c). It may help to sketch the triangle.
6. Using the formulas from #4, what is (a, b, c) if $(m, n) = (4, 1)$?
7. Consider these formulas: $a = m^2 - n^2$, $b = 2mn + n^2$, $c = m^2 + mn + n^2$. Find (a, b, c) if $(m, n) = (3, 1)$.
8. For the triangle found in the previous problem, use the law of cosines to find C .
9. Heron's formula: $A_{\Delta} = \sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$. A triangle with *rational side lengths* and *rational area* is a **Heronian** triangle. For the Heronian triangle to the right, first calculate s . Then use Heron's formula to find the area.
10. To *two* decimal places, find the measure of the obtuse angle in a 6-25-29 triangle.
11. Use Heron's formula to find the area of a 7-15-20 triangle.
12. To *two* decimal places, find the measure of the obtuse angle in a 7-15-20 triangle.
13. Find the area of a 3-25-26 triangle.
14. To *two* decimal places, find the measure of the larger acute angle in a 3-25-26 triangle. The law of cosines may be used, or the formula $A_{\Delta} = \frac{1}{2}ab \sin C$ may be used since the angle is acute and the area has been found. This is easier than using the law of cosines.
15. To *two* decimal places, find ϕ in the triangle to the right.
16. Find the area of a 3-5044-5045 triangle.
17. To *two* decimal places, find the measure of the larger acute angle in a 3-5044-5045 triangle. It may help to sketch the triangle.
18. Find the area of a 13-14-15 triangle.
19. Find the area of a 51-52-53 triangle.
20. To *two* decimal places, find θ in the triangle to the right.
21. Consider the bold italicized numbers in this list of Heronian triangles: $(3, \mathbf{4}, 5)$, $(13, \mathbf{14}, 15)$, $(51, \mathbf{52}, 53)$, $(193, \mathbf{194}, 195)$. Observe that $194 = 4 \cdot 52 - 14$. To find the next number in that sequence, quadruple the last number and subtract the previous number. Find the next *two* numbers after 194.
22. Consider the bold italicized numbers in these Heronian triangles: $(3, \mathbf{4}, 5)$, $(3, \mathbf{25}, 26)$, $(3, \mathbf{148}, 149)$, $(3, \mathbf{865}, 866)$. Observe that $148 = 6 \cdot 25 - 4 + 2$ and $865 = 6 \cdot 148 - 25 + 2$. To find the next number in the sequence, multiply the last number by 6 and subtract the previous number and add 2. Find the next *two* numbers after 865.
23. Evaluate $(5044 + 5045) \div 7134$ to *seven* decimal places.



Extra Credit: Find the *exact* lengths of the *three* altitudes for a 3-25-26 triangle. Hint: $A_{\Delta} = \frac{1}{2}bh$