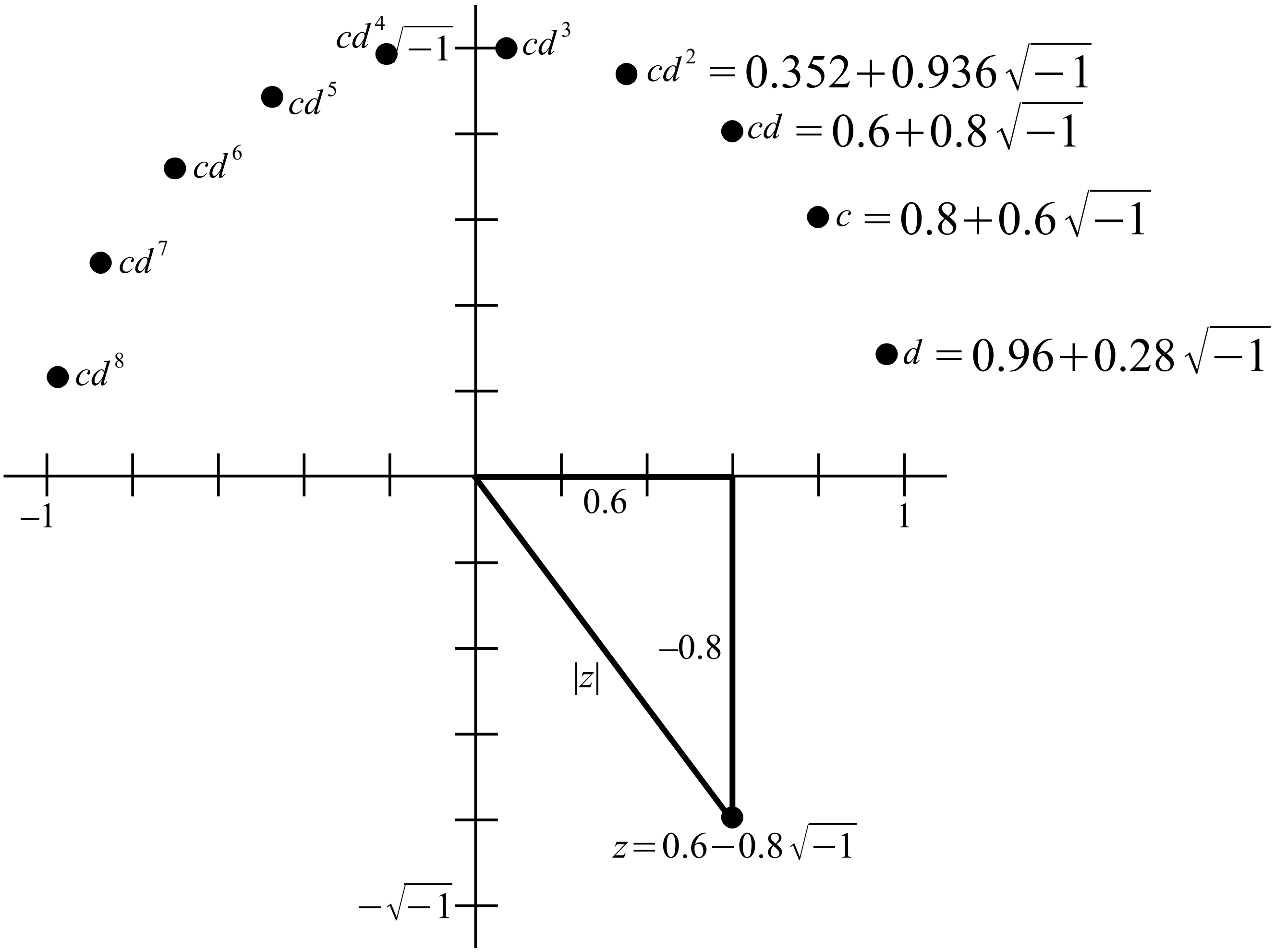
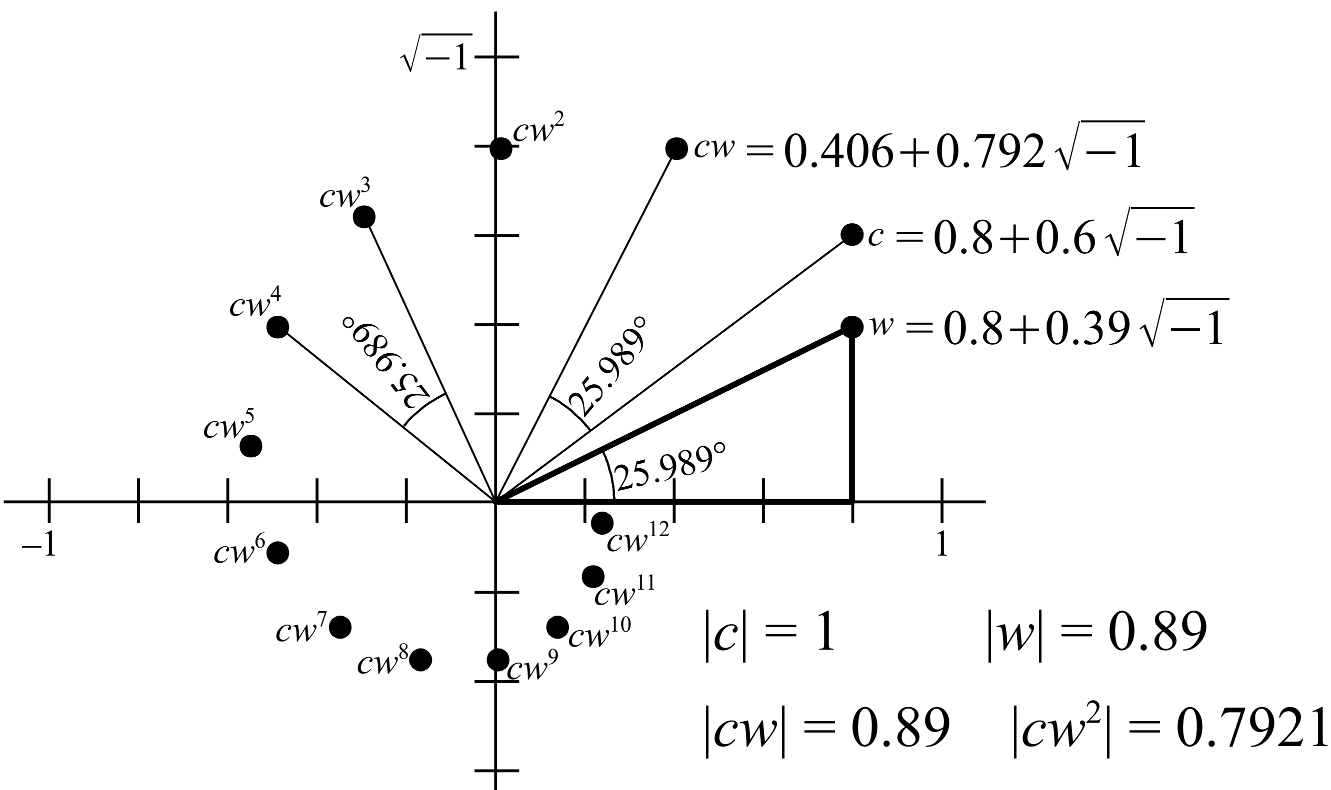


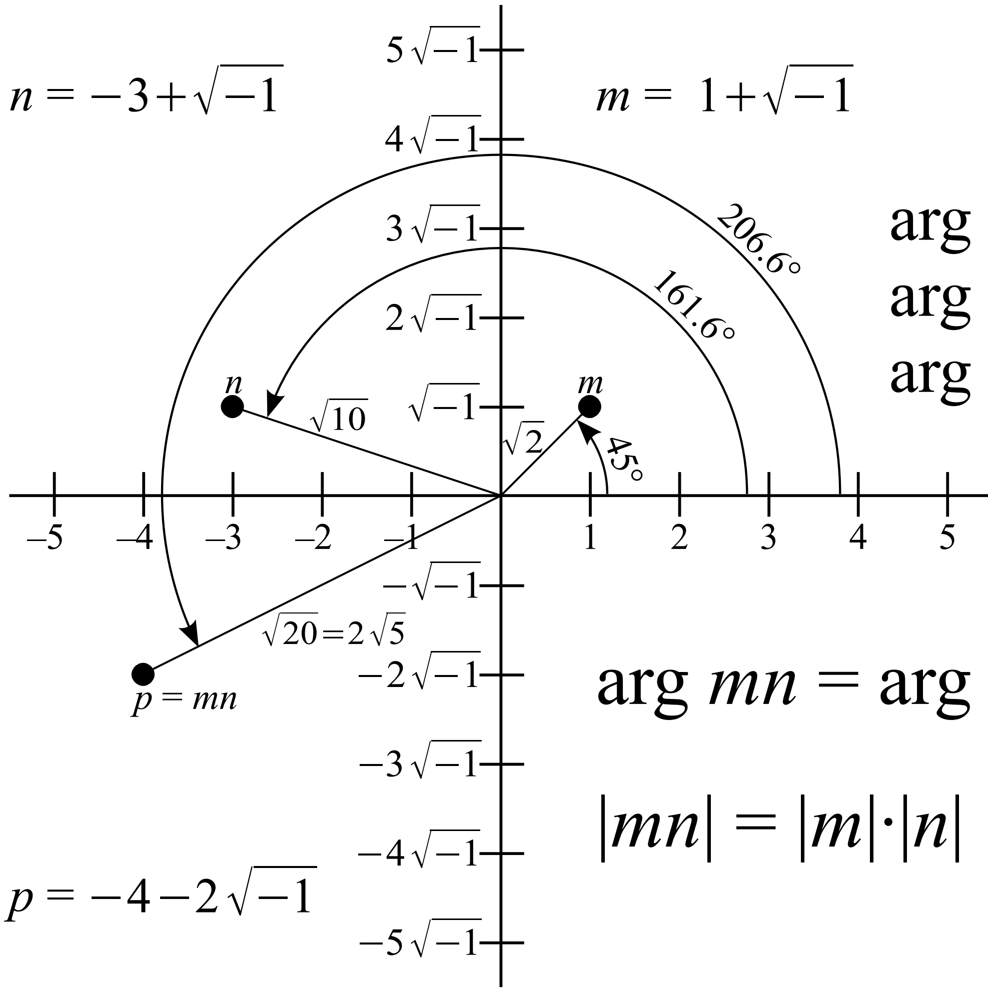


$|c| = 1 \qquad |w| = 0.89$

$|cw| = 0.89 \qquad |cw^2| = 0.7921$

$$|c \cdot w| = |c| \cdot |w|$$

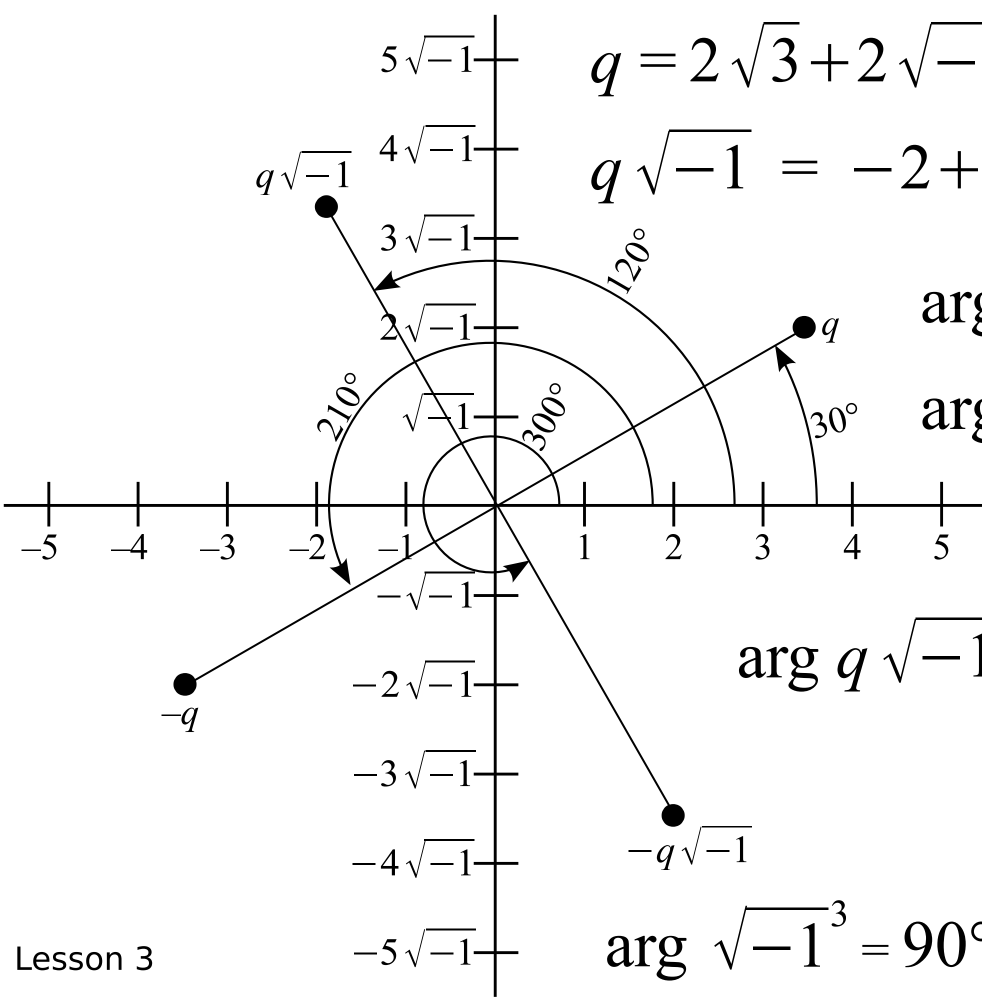
$$|c \cdot w \cdot w| = |c| \cdot |w| \cdot |w|$$



$$\arg m = 45°$$
$$\arg n \approx 161.6°$$
$$\arg mn \approx 206.6°$$

$$\arg mn = \arg m + \arg n$$

$$|mn| = |m| \cdot |n|$$



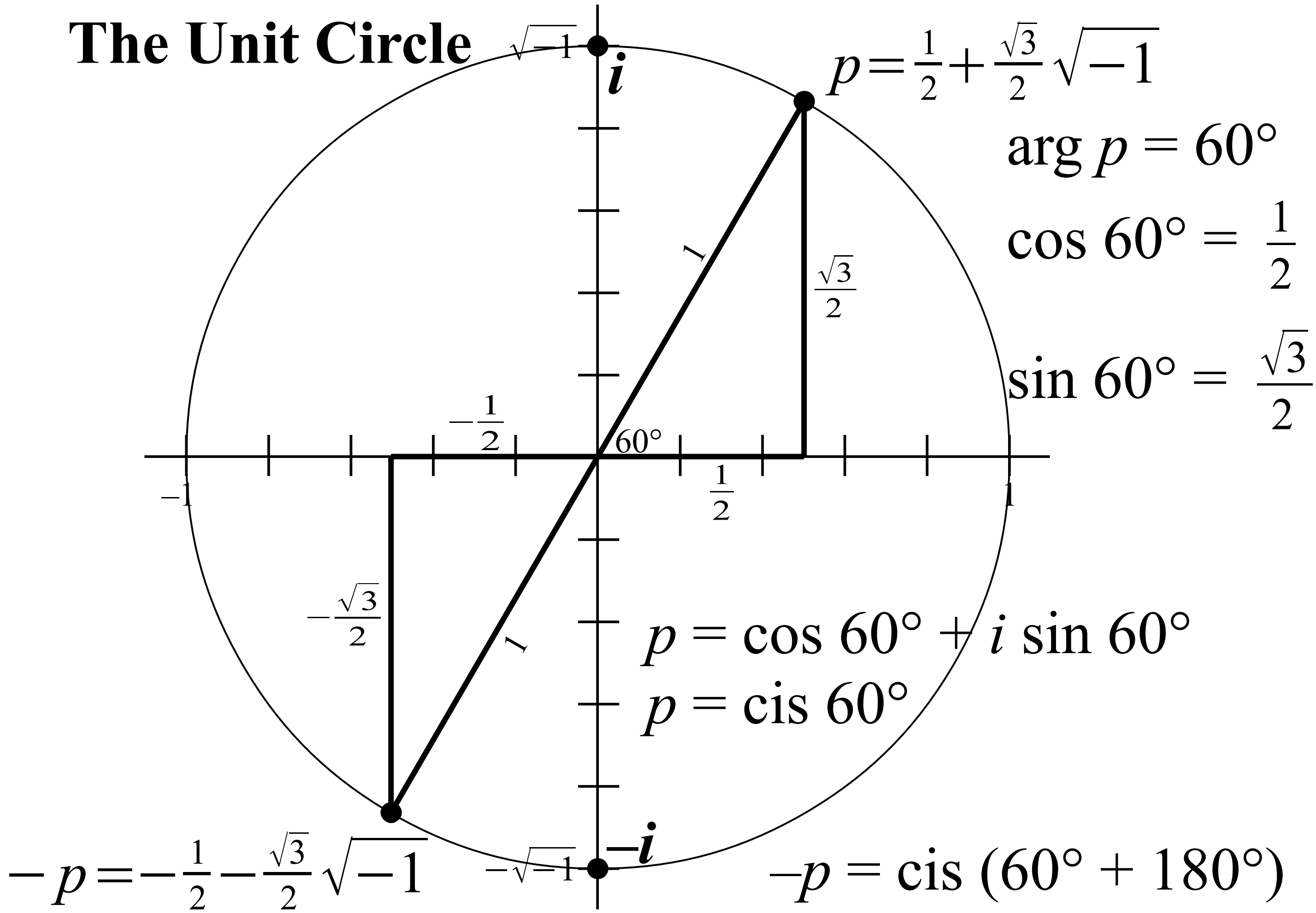
$$q = 2\sqrt{3}+2\sqrt{-1}$$
$$q\sqrt{-1} = -2+2\sqrt{3}\sqrt{-1}$$

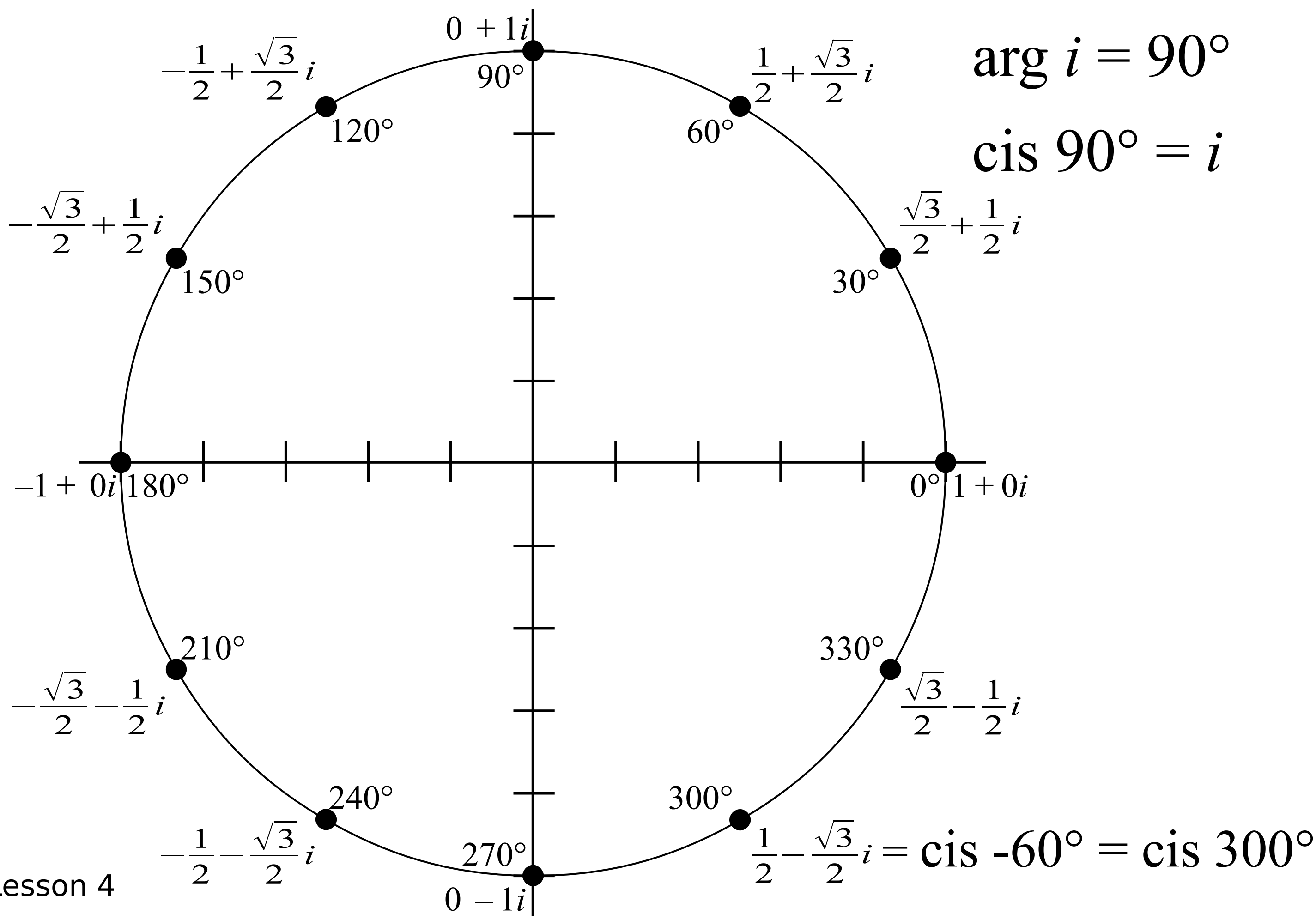
$$\arg q = 30°$$
$$\arg \sqrt{-1} = 90°$$

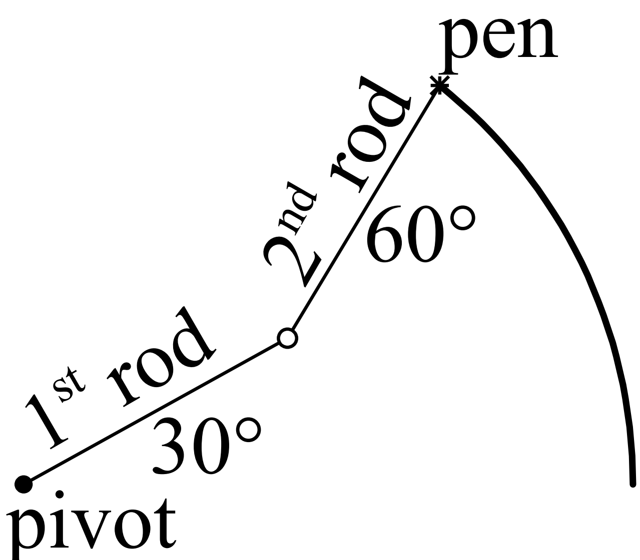
$$\arg q\sqrt{-1} = 30° + 90°$$

$$\arg \sqrt{-1}^3 = 90° + 90° + 90°$$

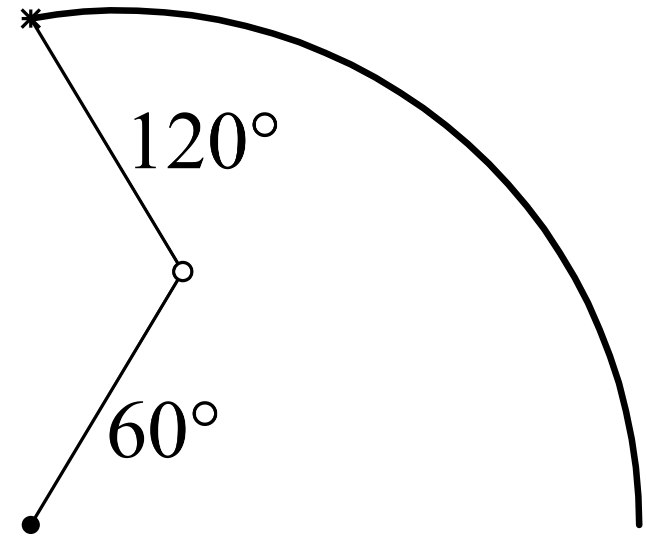
The Unit Circle





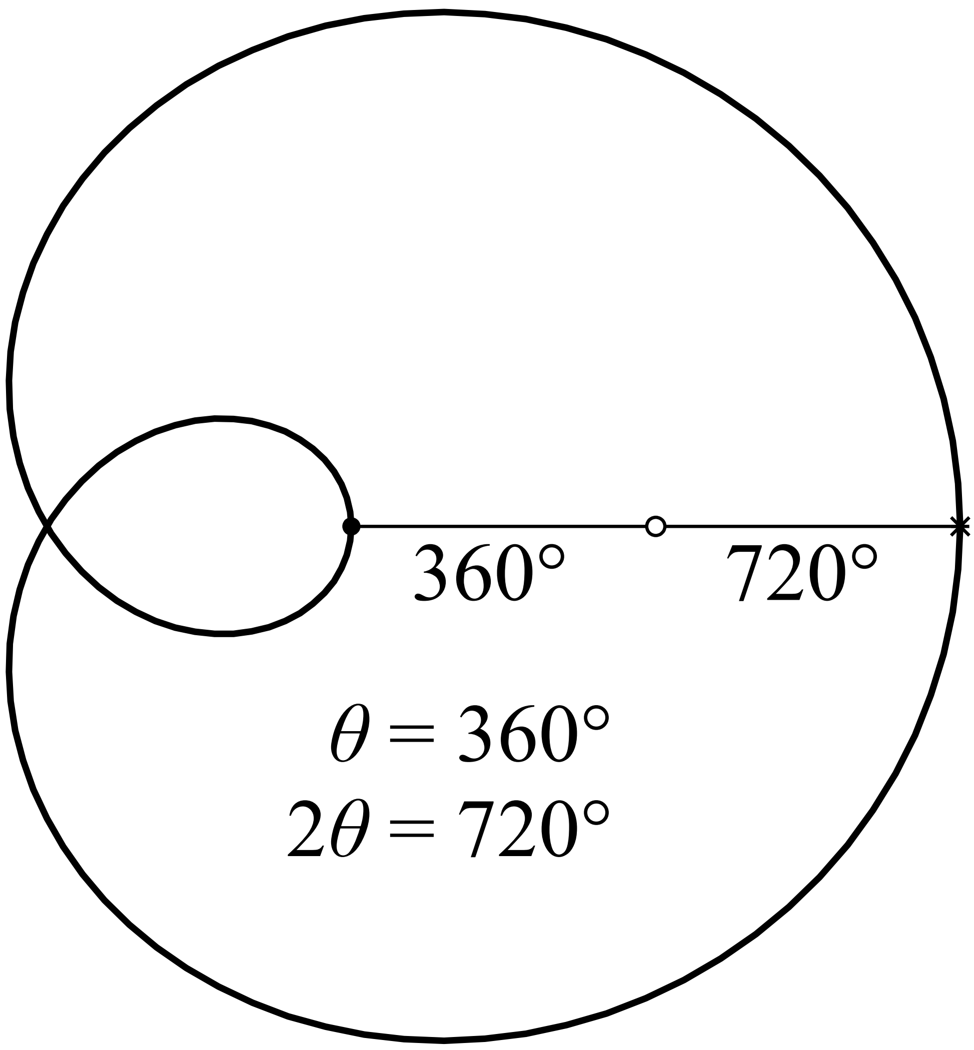


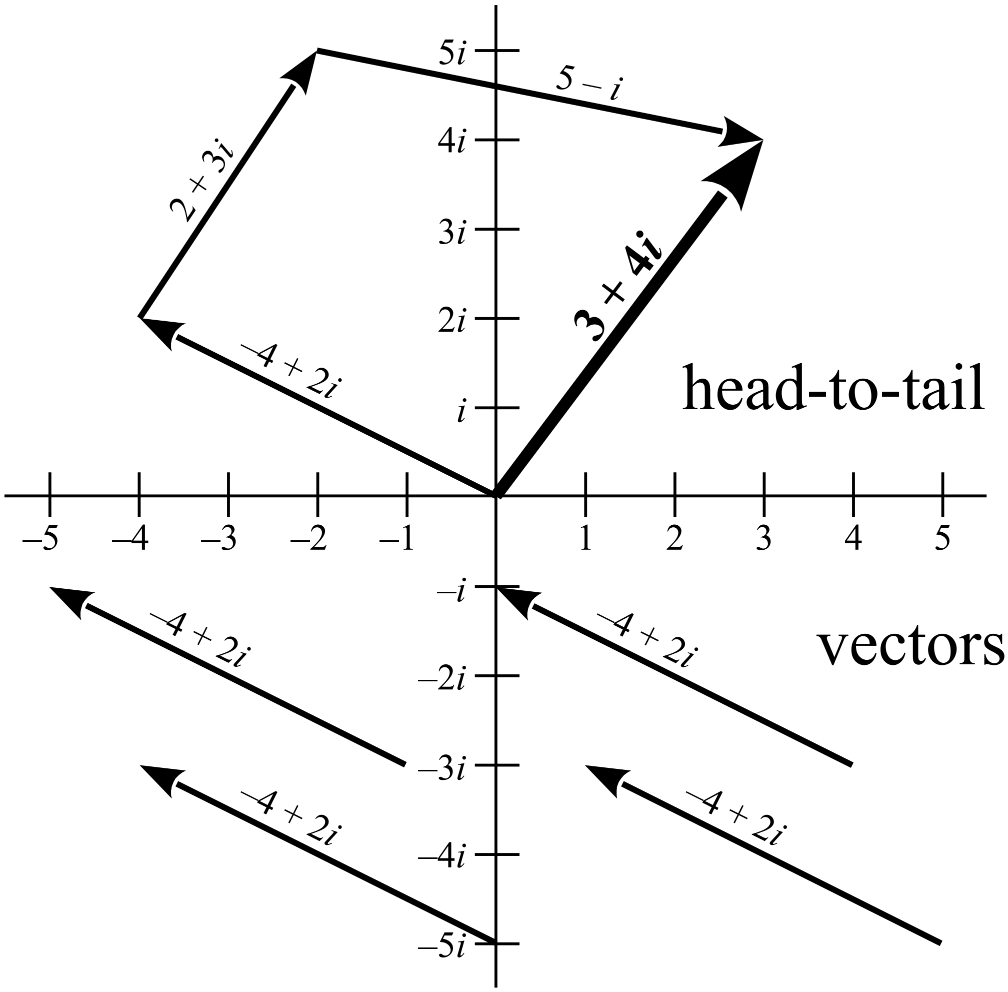
$\theta = 30°$
$2\theta = 60°$

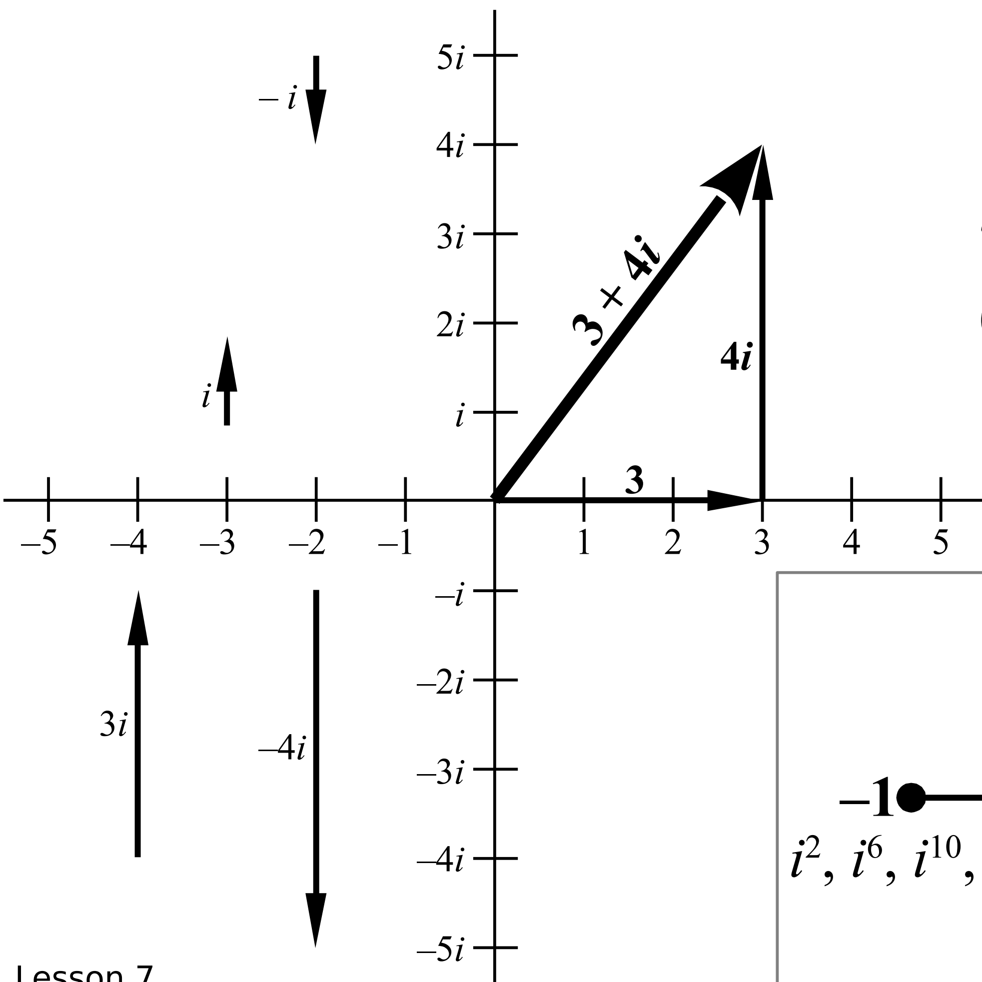


$\theta = 60°$
$2\theta = 120°$

$z = \text{cis}\ \theta + \text{cis}\ 2\theta$

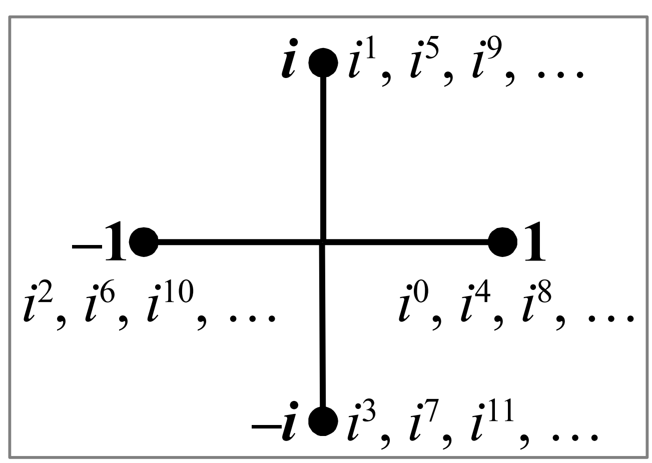


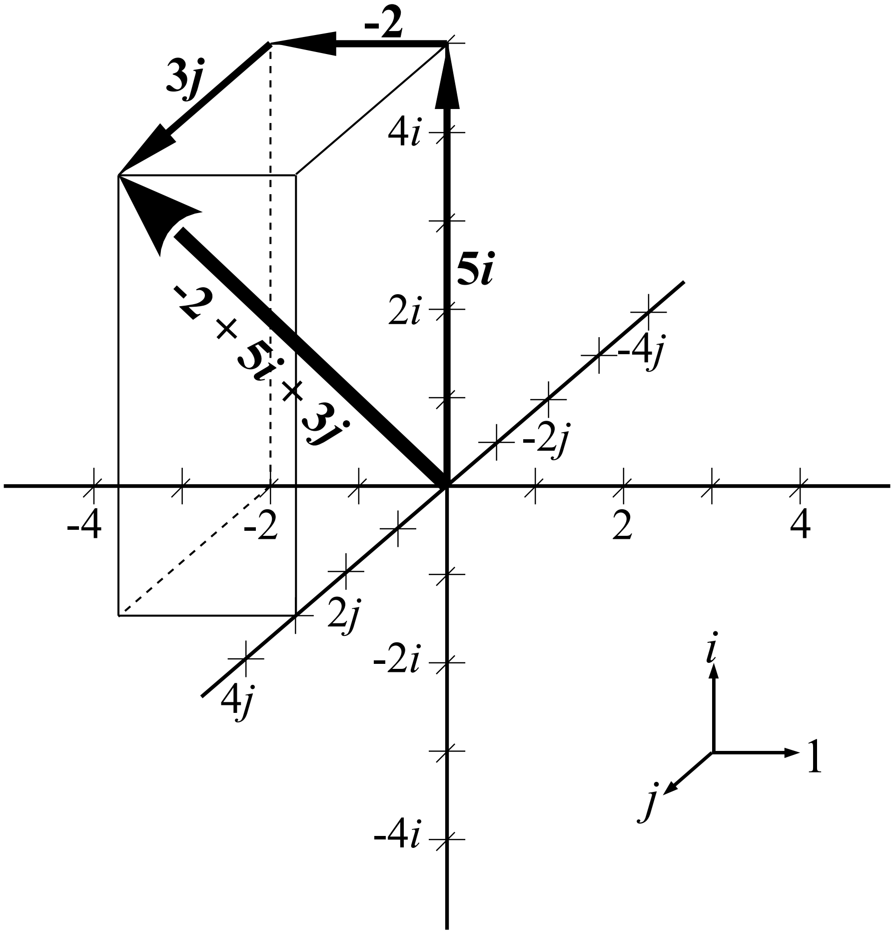

5i
4i
3i
2i
i
–i
–2i
–3i
–4i
–5i
–5
–4
–3
–2
–1
1
2
3
4
5
5 – i
2 + 3i
3 + 4i
–4 + 2i
head-to-tail
vectors


5i
4i
3i
2i
i
–i
–2i
–3i
–4i
–5i
–5
–4
–3
–2
–1
1
2
3
4
5
3 + 4i
4i
3
– i
i
3i
–4i

$$i^2 = -1$$

$$(-i)^2 = -1$$

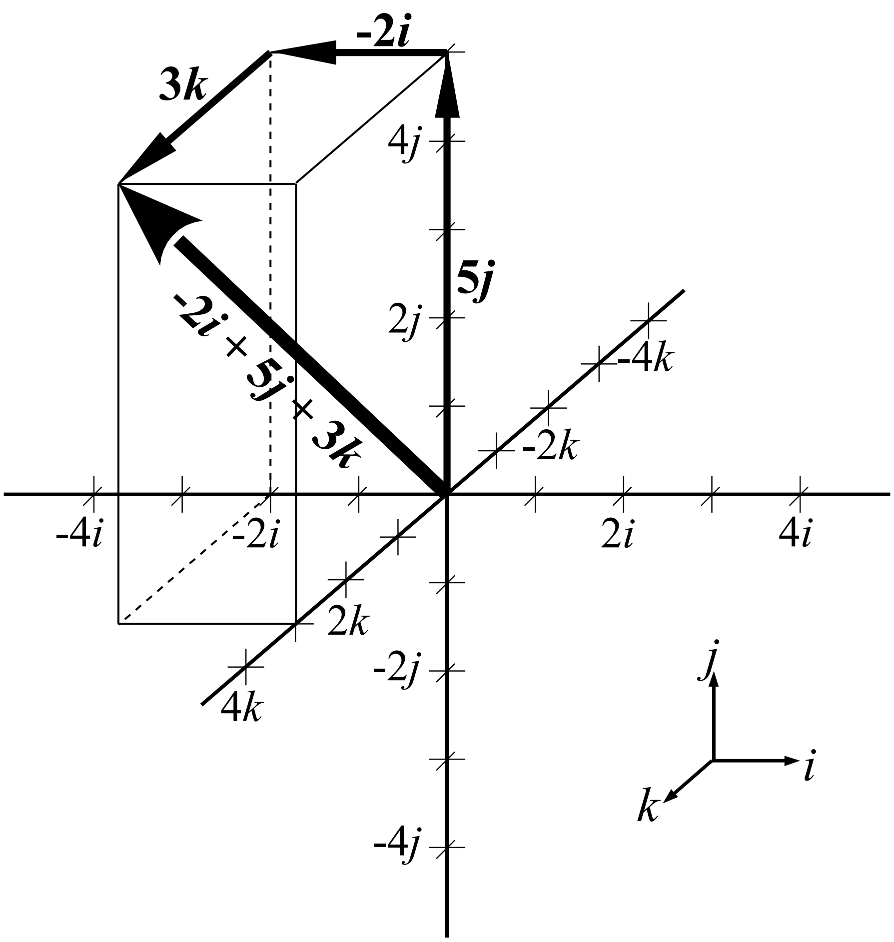

i
$i^1, i^5, i^9, \ldots$
–1
$i^2, i^6, i^{10}, \ldots$
1
$i^0, i^4, i^8, \ldots$
–i
$i^3, i^7, i^{11}, \ldots$



$$i^2 = -1$$

$$j^2 = -1$$

$$k^2 = -1$$

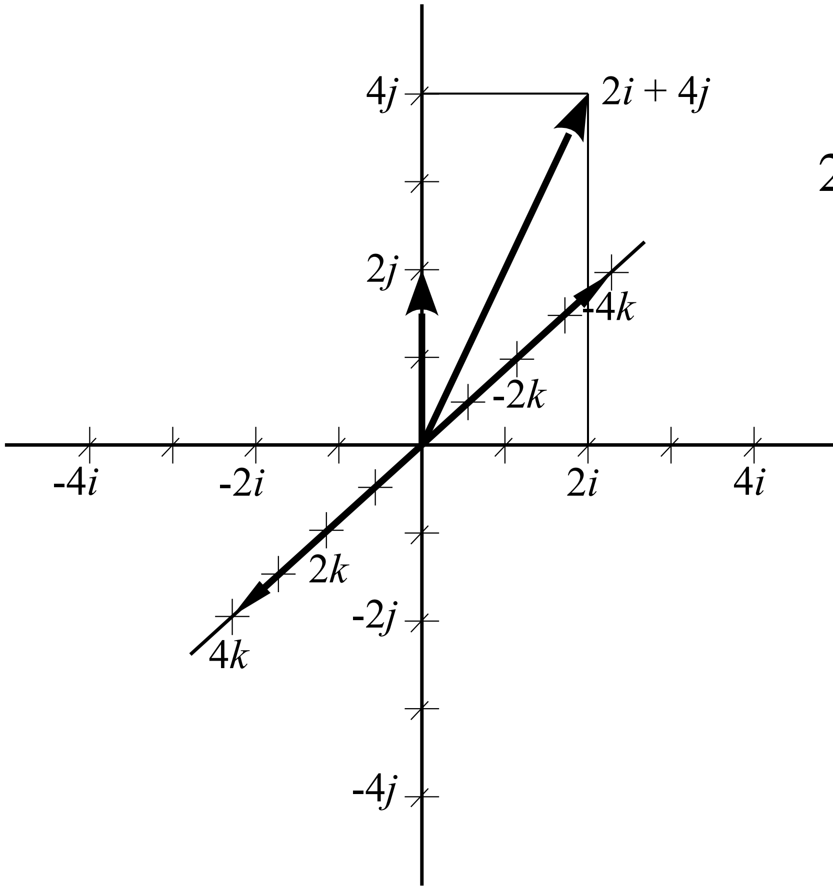


$$ij = k \qquad ji = -k$$

$$jk = i \qquad kj = -i$$

$$ki = j \qquad ik = -j$$

3-D Vectors



$2\mathbf{i} + 4\mathbf{j}$

$2\vec{\imath} + 4\vec{\jmath}$

$(2i + 4j)(2j)$	$(2j)(2i + 4j)$	Multiply 2 ways
$4ij + 8j^2$	$4ji + 8j^2$	Distribute
$4k + 8(-1)$	$4(-k) + 8(-1)$	$ij = k$ and $ji = -k$ and $j^2 = -1$
$-8 + 4k$	$-8 - 4k$	Simplify and rearrange

2 cross products: $(2i + 4j)\times(2j) = 4k$ and $(2j)\times(2i + 4j) = -4k$

1 dot product: $(2i + 4j)\bullet(2j) = 8$ or $(2j)\bullet(2i + 4j) = 8$

z^2

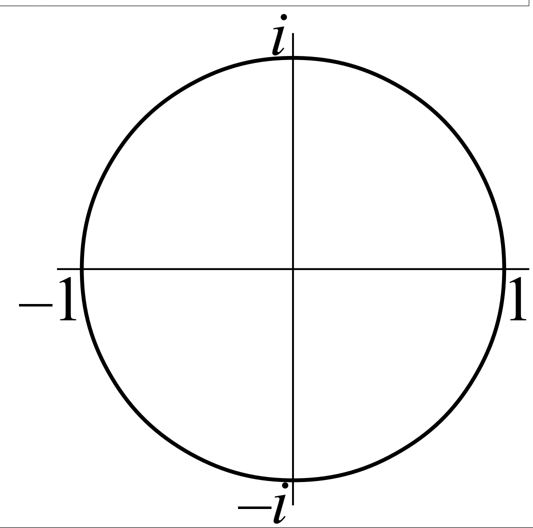
z_0	i	$2i$	$1.1i$	$0.9i$	cis 30°	$1 + i$
z_1	–1	–4	–1.21	–0.81	cis 60°	$2i$
z_2	1	16	1.4641	0.6561	cis 120°	–4
z_3	1	256	2.1436	0.4305	cis 240°	16
z_4	1	65,536	4.5950	0.1853	cis 120°	256

Orbit for $z_0 = 2i$: $\{2i, –4, 16, 256, ...\}$

Attracted to infinity

Orbit for $z_0 = i$: $\{i, –1, 1, 1, \ldots\}$

Not attracted to infinity



Gaston Julia: Boundary of numbers attracted to infinity

i

$z^2 – 1$

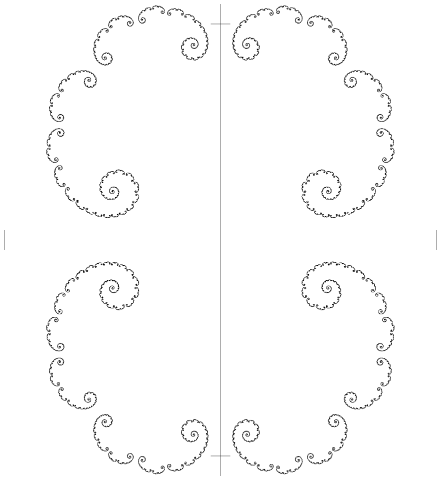
–1 1

$–i$

$$z^2 - 0.74 + 0.08i$$

$$z^2 + 0.26 + 0.5i$$

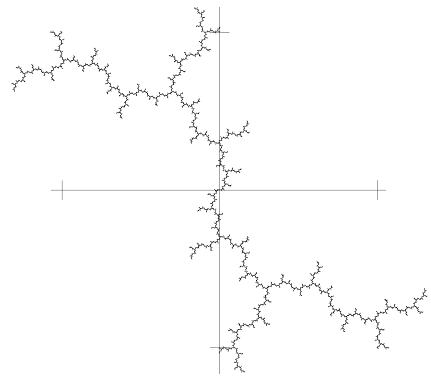
$z^2 + 0.3$



Disconnected

$c = 0.3$

$z^2 + i$

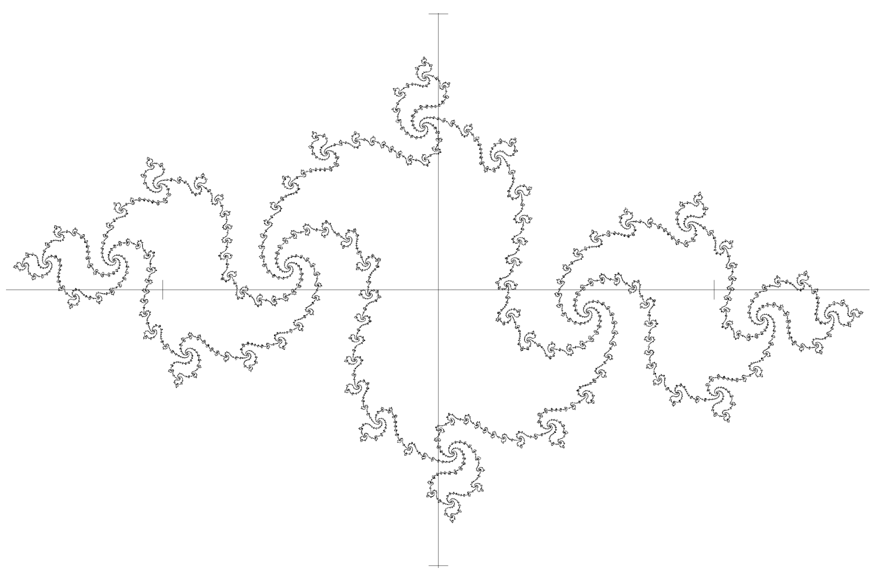


Connected

$c = i$

$z^2 + c$

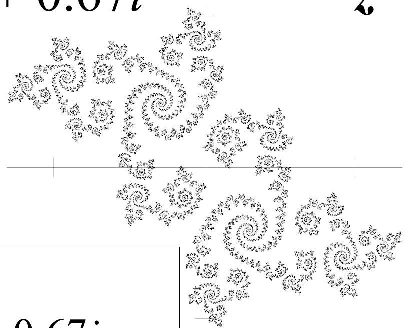
$z^2 - 0.83 + 0.17i$

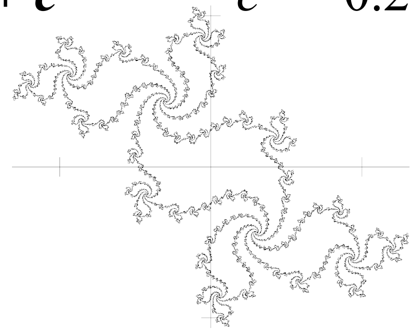


Connected

$c = -0.83 + 0.17i$

$c = -0.2 + 0.67i$ $\qquad$ $\boldsymbol{z^2 + c}$ $\qquad$ $c = -0.2 + 0.7i$





z_0	$=$	0
z_1	$=$	$-0.2 + 0.67i$
z_2	$\approx$	$-0.61 + 0.40i$
z_3	$\approx$	$0.01 + 0.18i$
	$\vdots$	
z_{25}	$\approx$	$-1.97 + 3.80i$
z_{26}	$\approx$	$-10.79 - 14.33i$

In these lessons, $|c| \le 2$.

$|-1.97 + 3.80i| \approx 4.28 \ge 2$

z_0	$=$	0
z_1	$=$	$-0.2 + 0.7i$
z_2	$=$	$-0.65 + 0.42i$
z_3	$\approx$	$0.05 + 0.15i$
	$\vdots$	
z_{201}	$\approx$	$0.13 + 0.14i$
z_{202}	$\approx$	$-0.20 + 0.74i$
z_{203}	$\approx$	$-0.70 + 0.40i$
z_{204}	$\approx$	$0.13 + 0.14i$
z_{205}	$\approx$	$-0.20 + 0.74i$
z_{206}	$\approx$	$-0.70 + 0.40i$

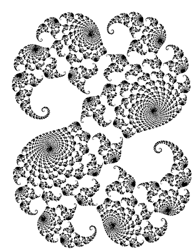
Identify the Julia set with the given value of c.

$c = 0.28 + 0.02i$

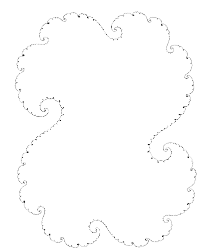
$z_0 = 0$

z_{95}	$\approx$	$0.44 + 0.19i$
z_{96}	$\approx$	$0.44 + 0.18i$
z_{97}	$\approx$	$0.44 + 0.18i$
z_{98}	$\approx$	$0.44 + 0.18i$
z_{99}	$\approx$	$0.44 + 0.18i$

A)



B)



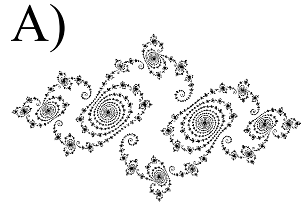
For the other Julia set, $c = 0.28 + 0.01i$.

$c = -0.75 + 0.11i$

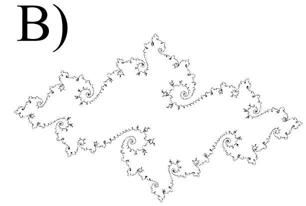
$z_0 = 0$

z_{25}	$\approx$	$-0.36 + 0.69i$
z_{26}	$\approx$	$-1.09 - 0.39i$
z_{27}	$\approx$	$0.28 + 0.96i$
z_{28}	$\approx$	$-1.58 + 0.65i$
z_{29}	$\approx$	$1.33 - 1.95i$

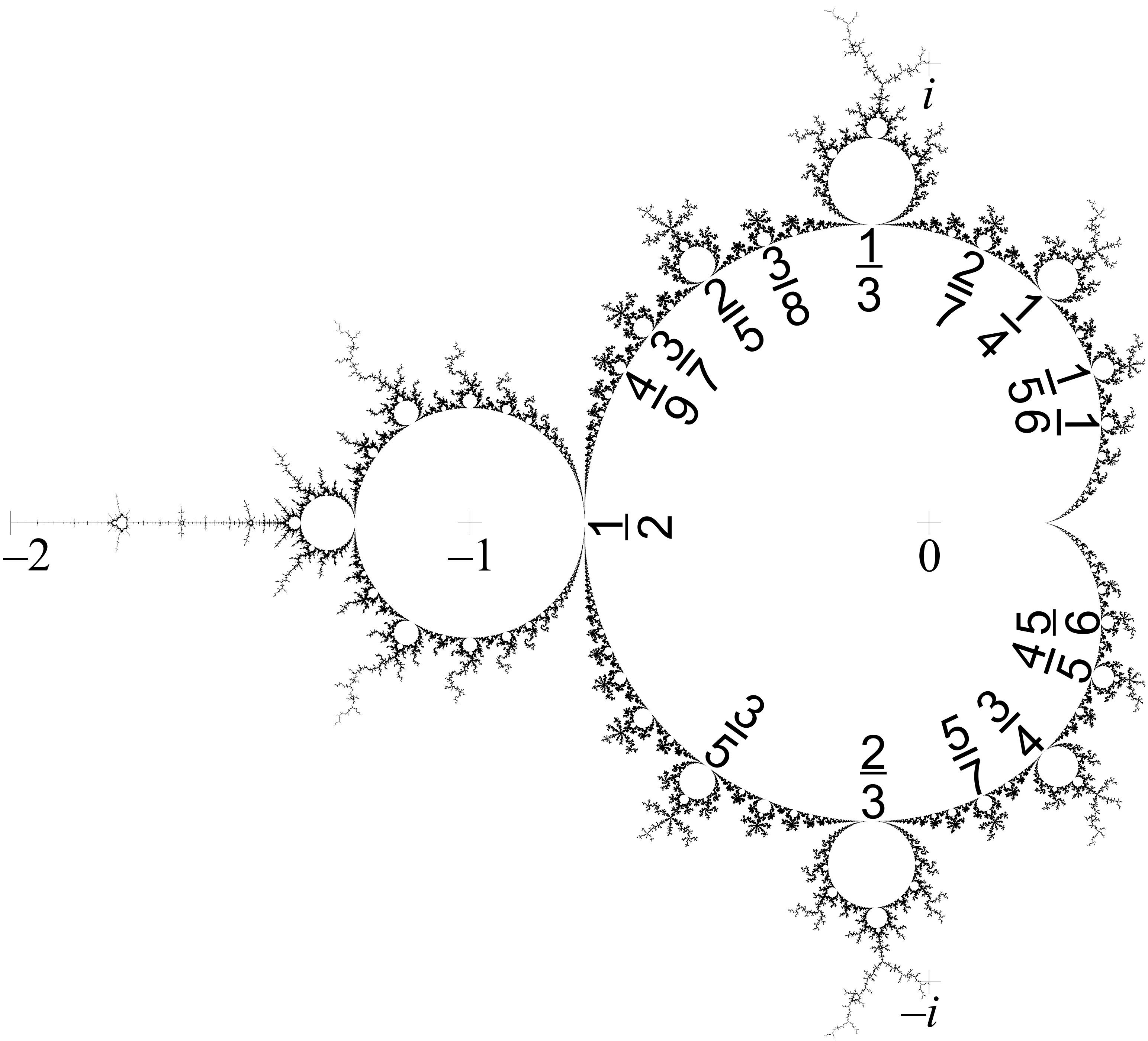
A)



B)

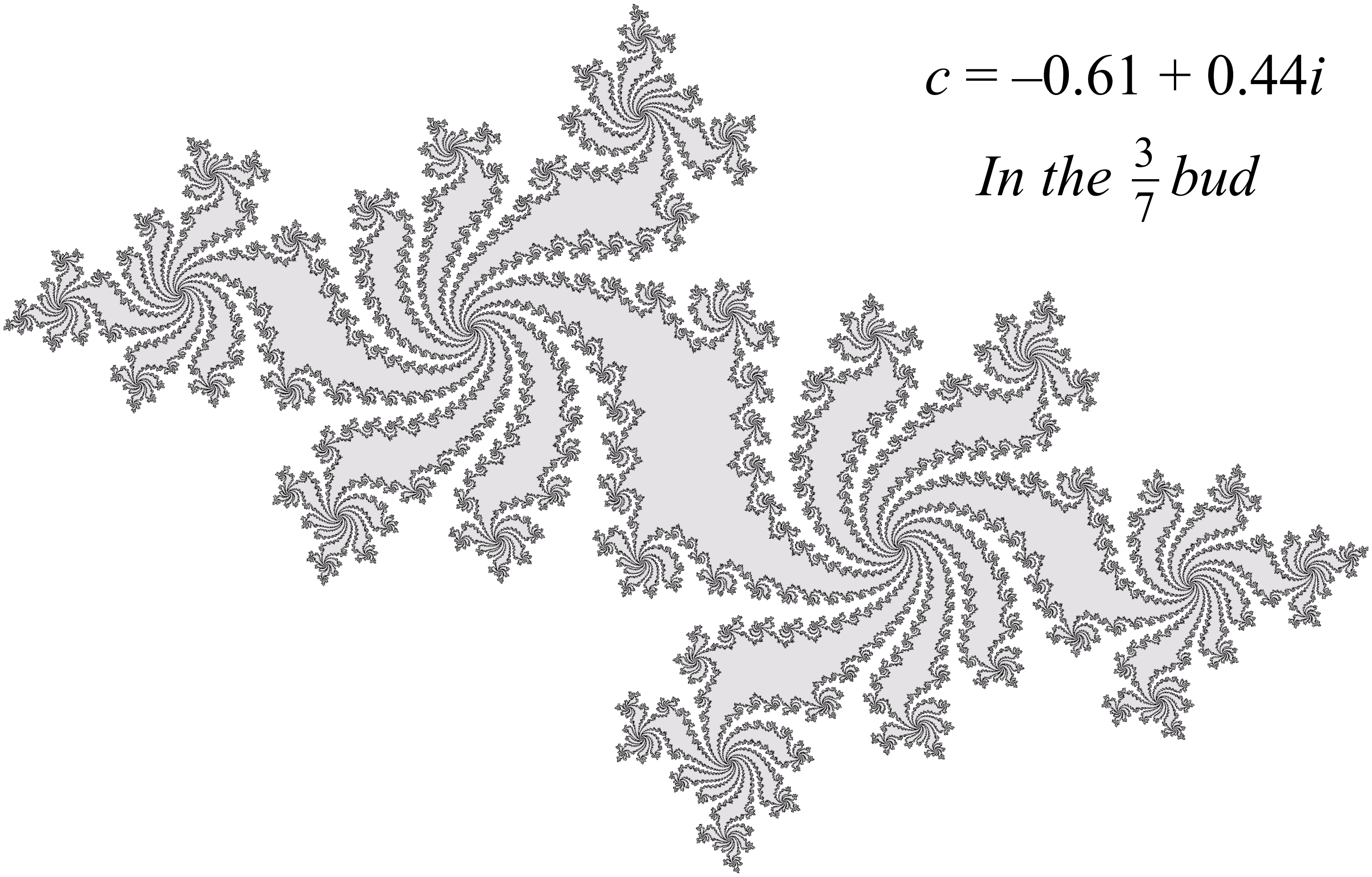


For the other Julia set, $c = -0.74 + 0.11i$.



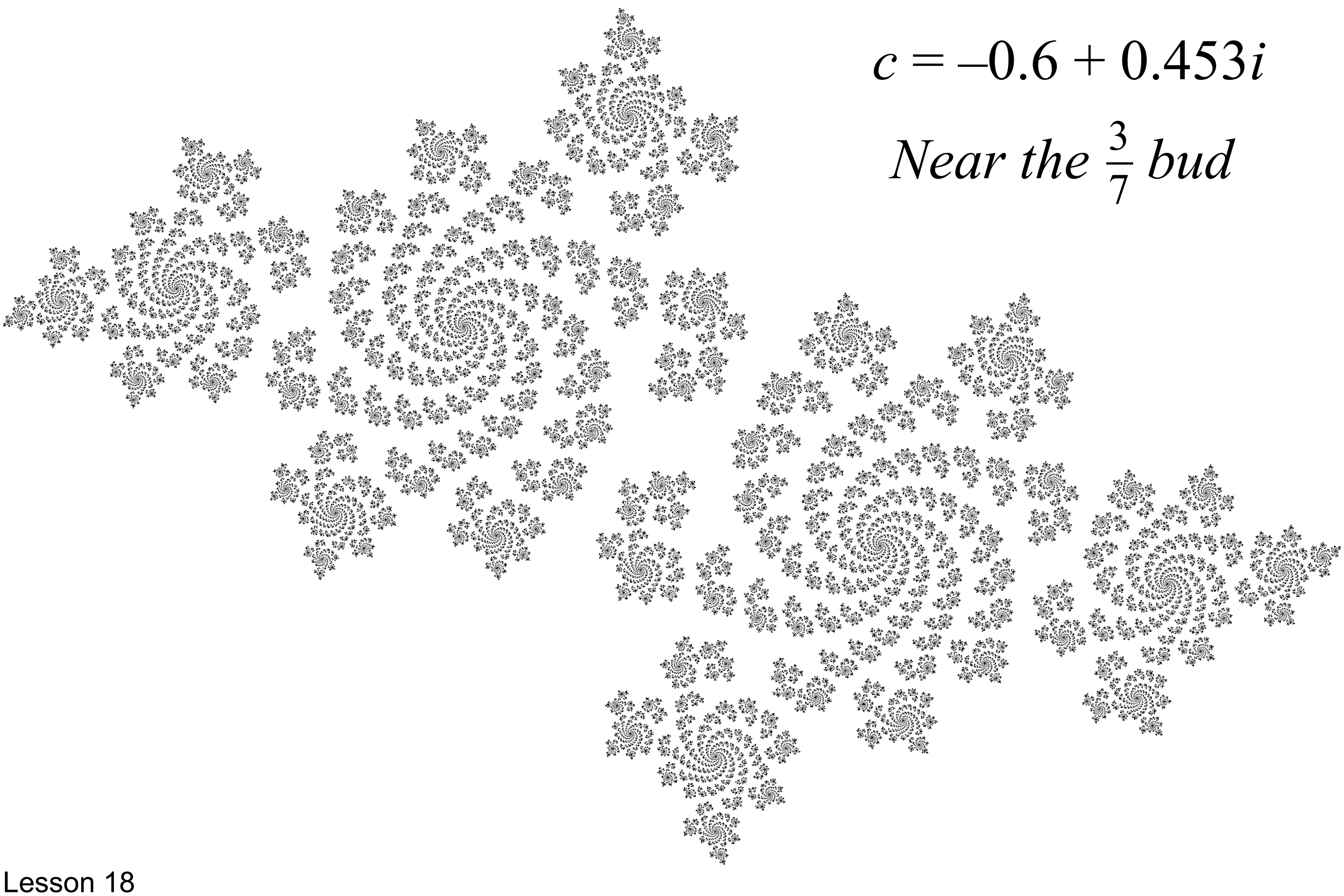
Benoit B. Mandelbrot: Map of connected Julia sets

Cardioid: $\boldsymbol{c = \frac{1}{2}\,\mathrm{cis}\,\theta - \frac{1}{4}\,\mathrm{cis}\,2\theta}$



$c = -0.61 + 0.44i$

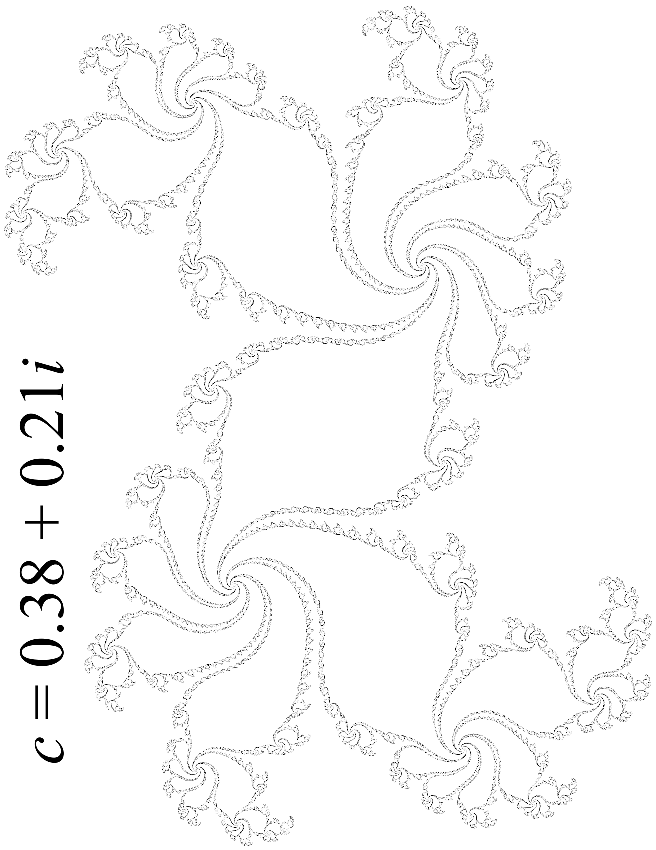
In the $\frac{3}{7}$ *bud*



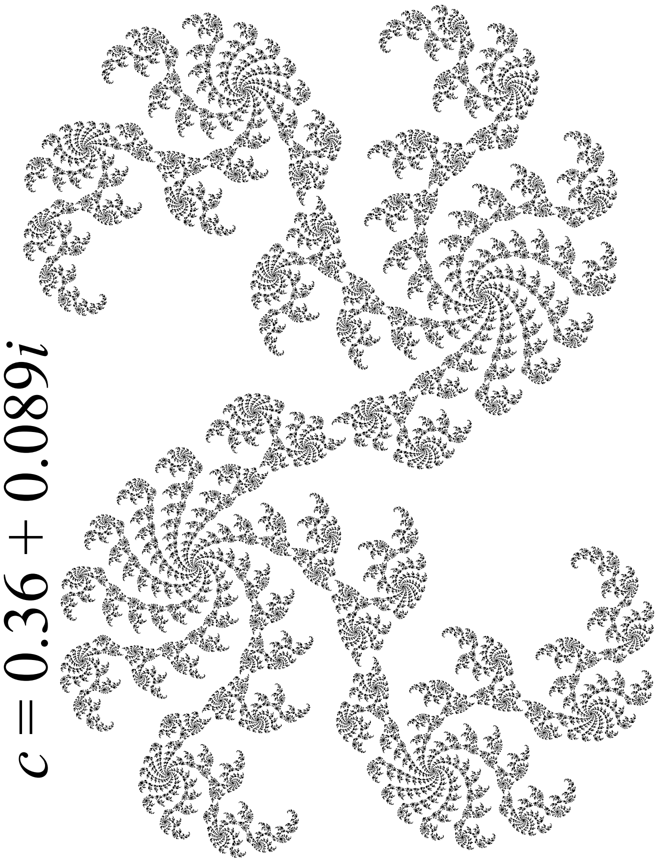
$c = -0.6 + 0.453i$

Near the $\frac{3}{7}$ *bud*

What is the denominator of the fraction for the bud associated with each Julia set?



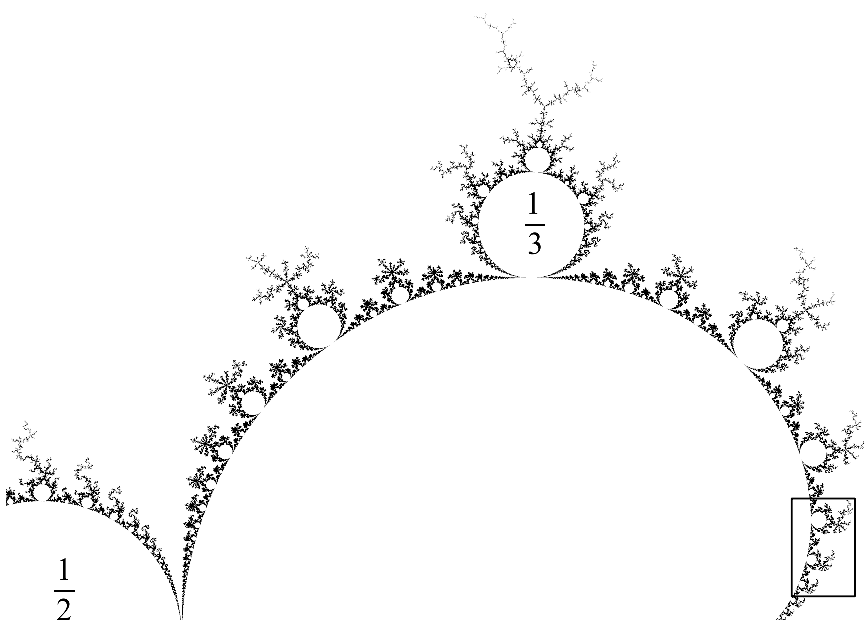
$c = 0.38 + 0.21i$

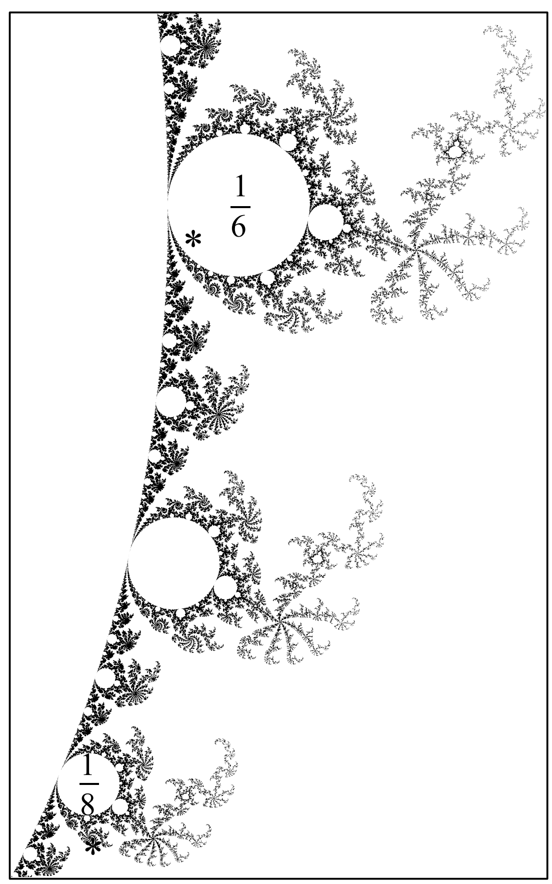


$c = 0.36 + 0.089i$

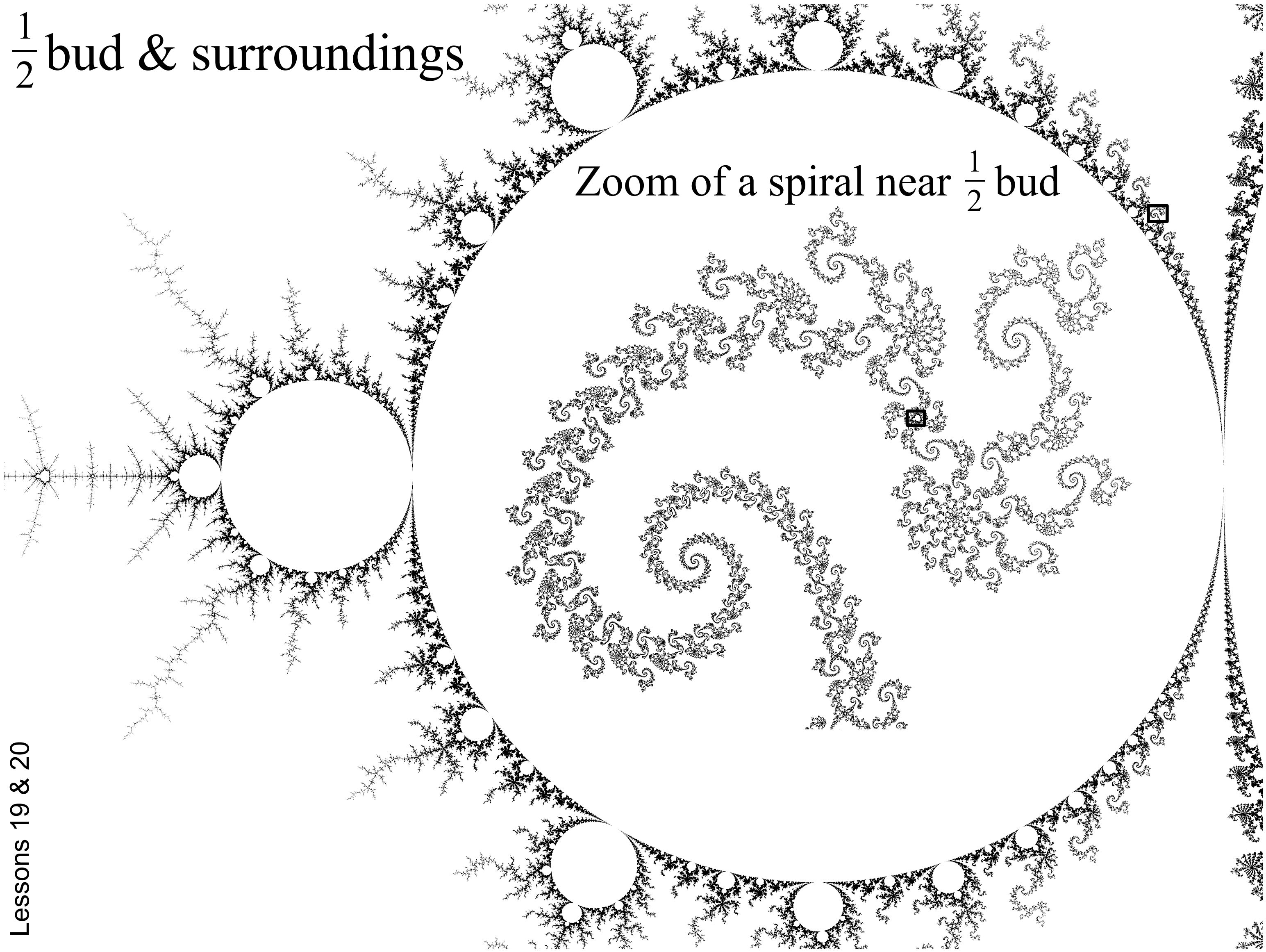
Left: 6 Right: 8

Asterisks show c locations:



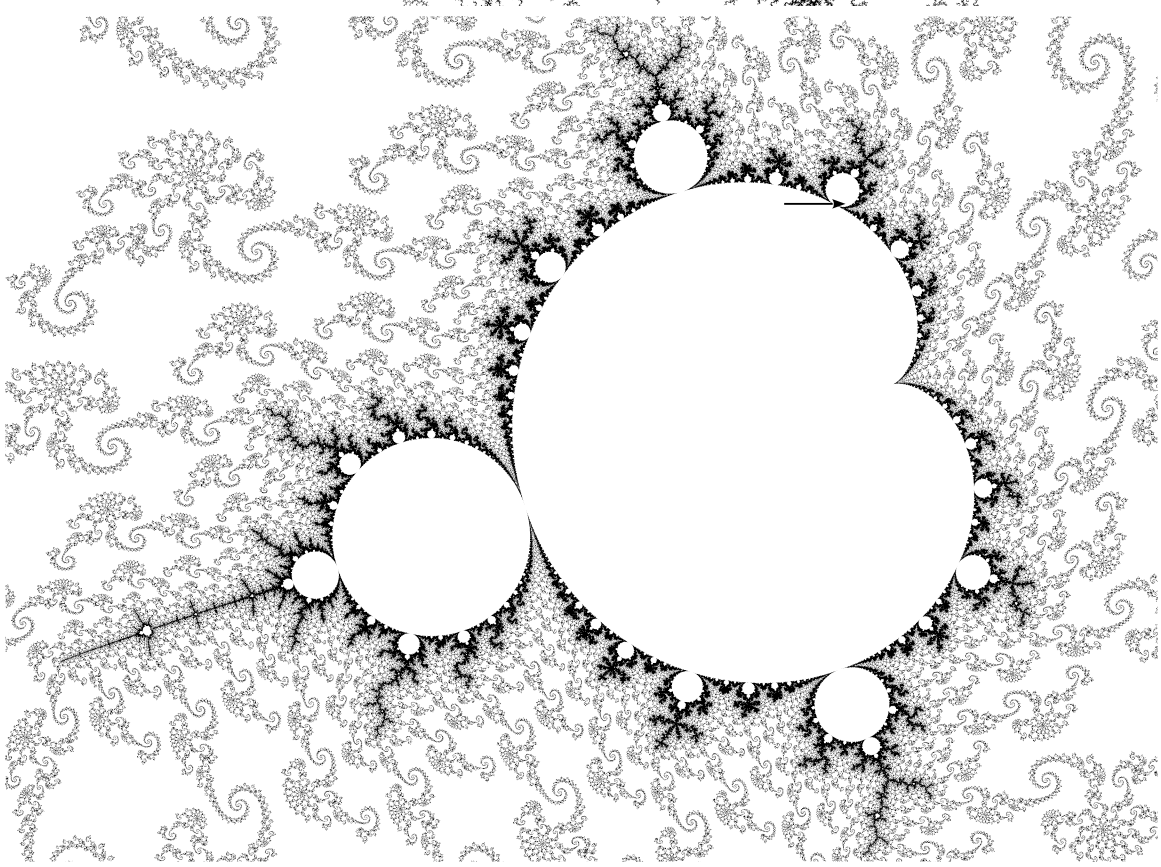


$\frac{1}{2}$ bud & surroundings



Zoom of a spiral near $\frac{1}{2}$ bud

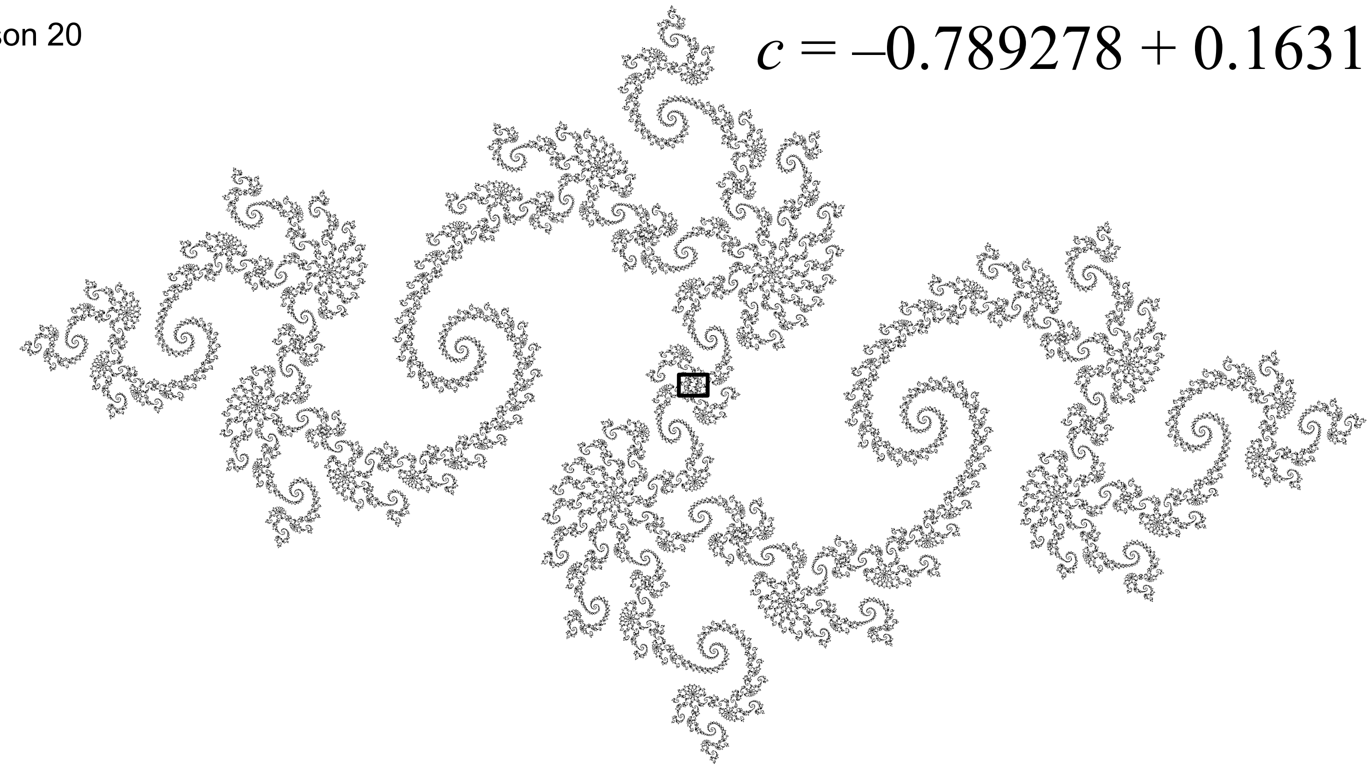
Lessons 19 & 20



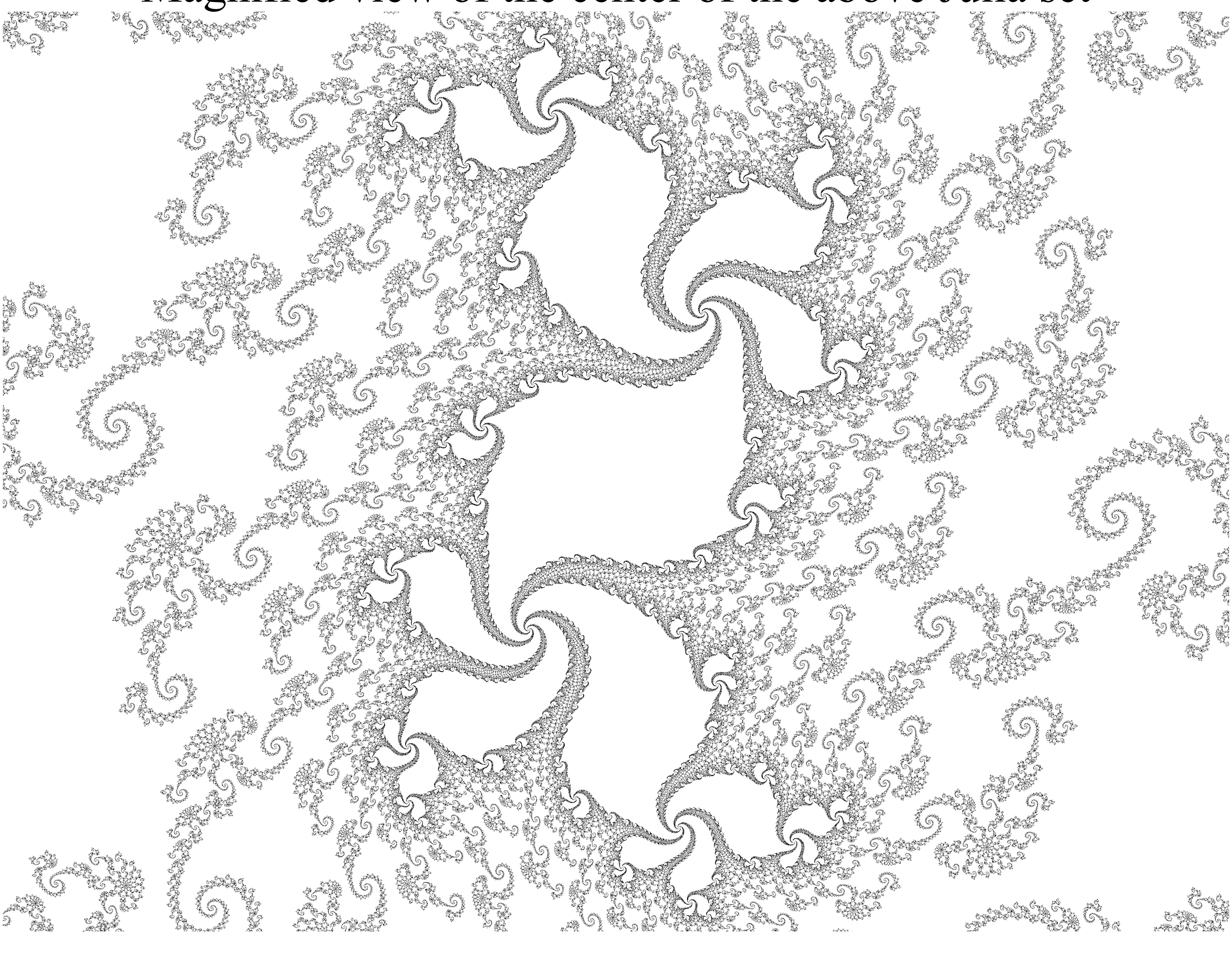
This miniature Mandelbrot
is a small part of the spiral

Arrow points into $\frac{1}{4}$ bud to
$c = -0.789278 + 0.163134i$

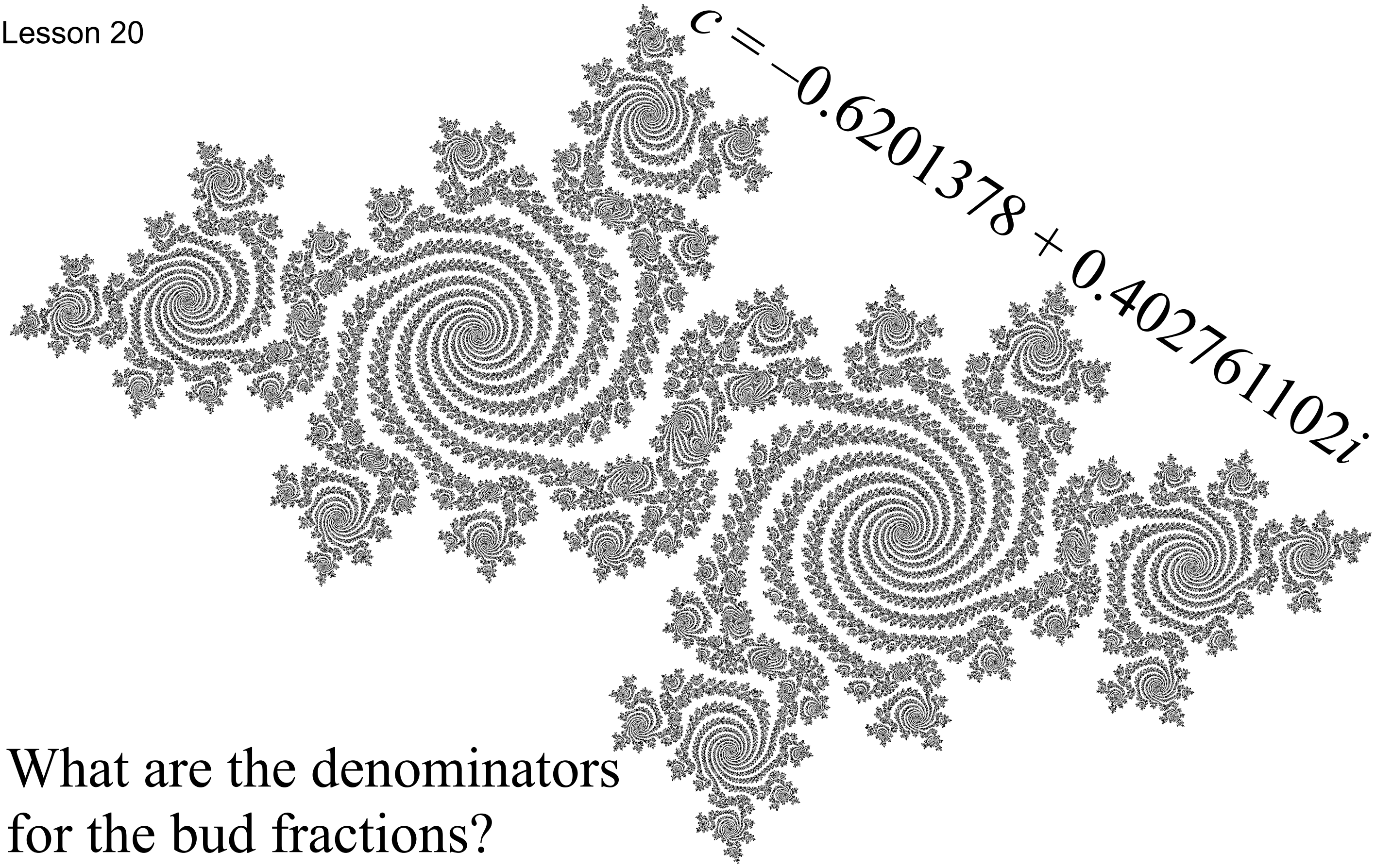
$c = -0.789278 + 0.163134i$



Magnified view of the center of the above Julia set



$c = -0.6201378 + 0.4027761102i$



What are the denominators for the bud fractions?